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THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

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THE LAW OF PHOTO-ELECTRIC PHOTOMETRY

By JAKOB KUNZ

In the field of present photo-electric investigations four main groups may be distinguished. The first group is concerned with the fundamental problem of the velocity of electrons emitted, and the frequency of the incident light. After a long series of experiments of gradually increasing accuracy the equation $\frac{1}{2}mv^2=hn-P_0$ has finally been established by the measurements of R. A. Millikan. The second problem is that of the relation between the number of electrons emitted and the intensity of the incident light. Other problems arise in connection with the so-called normal and selective effects, and with the influence of gases. The present investigation deals with the second problem. Several physicists have attacked this question with widely discordant results or conclusions. Lenard, Elster and Geitel, and Richtmyer concluded that a linear relation exists between the photo-electric current and the intensity of light. Griffith and H. E. Ives, on the other hand, do not find a linear relation. Ives especially[1] gives a large variety of curves and suggests a cell which will yield a straight line, without giving data for this statement.

[1] *Astrophysical Journal*, **39**, 428, 1914; **43**, 9, 1916.

A closer investigation of the literature, however, shows that there is very little if any evidence in favor of the simple, straight-line relation. Ives has already remarked that Lenard was not justified in his conclusion, as his figures really show a current increasing more rapidly than the illumination. Richtmyer also concluded from his data that the action of light is strictly proportional to the light-intensity over very wide ranges. If I divide, however, the relative light-intensities I by the deflections d, then the ratio I/d varies widely, as will be seen from Table I.

TABLE I

I	d	I/d	I	i	I/i
0.0100	0.318	314	19.0	5.9	322
0.0236	0.646	370	30.0	8.7	345
0.0400	1.033	389	45.0	12.5	360
0.0625	1.573	398	63.0	16.4	385
0.0816	2.055	397	84.0	22.6	372
0.111	2.682	414	118.0	31.4	376
0.131	3.20	409	155.0	38.8	398
0.160	3.93	407	210.0	52.6	400
0.197	4.80	410	304.0	79.5	383
0.297	7.51	396	375.0	97.5	385
0.387	9.79	396	475.0	123.0	386
0.760	18.53	411	620.0	160.0	387

The ratios I/i and I/d vary so much that the conclusion seems hardly justified that the photo-electric current is proportional to the intensity of illumination. Richtmyer has given further evidence in favor of his conclusion with experiments involving a photo-electric cell itself as a resistance. I have repeated these measurements with the same result, using cells, however, which do not give a straight-line relation when tested with different methods. Elster and Geitel have made photometric measurements also over a very wide range of intensities, using the polarization of light by a Nicol prism for the variation of the illumination. They also conclude that there is a linear relation between the two variables. In each set of experiments, however, they vary the intensity only in the ratio of 1 to 4, by means of the angles 30°, 45°, 60°, 90°, the deviations from the straight line varying only between 0.1 and 3 per cent. I was able to test an Elster and Geitel cell over a wider range of intensities, keeping the potential-difference con-

stant. Table II contains the results, d being the corrected deflections of the electrometer needle, I the relative intensity, I/d their ratio, which ought to be constant.

TABLE II

d	I	I/d	d	I	I/d
2.0	4	200	115.3	178	155
4.55	9	198	152.6	224	147
8.1	16	198	208.4	294	141
19.35	36	186	261.5	352	135
58.25	100	162			
90.25	144	160	310.85	400	130

The source of light was a tungsten lamp of 42 candles, which was moved over an interval between 30 and 300 cm from the sensitive surface. If the relative intensity of illumination varies from 4 to 16, then the ratio I/d varies, indeed, only 1 per cent, but if the intensity varies more, then the deviations from the straight line become conspicuous. Similar tests on other cells show clearly that the statements made by Ives are justified. Ives suggested, in his last contribution made in this journal, a cell which would give a straight line. Such a cell has been constructed according to the specifications given by Ives. Table III does show a very nearly constant ratio between the light-intensity and the photo-electric current.

TABLE III

d	I	I/d	d	I	I/d
3.45	4	116.0	128.1	144	112.5
8.0	9	112.5	159.4	178	112.0
14.1	16	114.0	202.6	224	111.6
31.6	36	114.0	257.6	294	114.0
86.4	100	116.0	306.7	352	114.5

This cell indeed satisfies very nearly the theoretical requirements of photometry. However, the light-intensities which act on the sensitive potassium surface are comparatively small and other cells of the older type also give a nearly linear relation for smaller light-intensities. A milk glass has almost necessarily to be used in connection with this cell, and thereby the intensities acting on

the surface are rather small. The electrostatic field in this cell is not uniformly radial, because it is well-nigh impossible to distil the alkali metal away from the glass stem which carries the central bulb covered with the potassium. This potassium cell contained argon. With the exception of this cell of Ives, it seems to me that there is no satisfactory evidence in favor of the single law of direct proportionality between the intensity of illumination and the photo-electric current. This situation gave rise to the following investigation.

During our study of the best conditions of the sensitiveness of photo-electric cells for the use of stellar photometry, Professor J. Stebbins and I found that in order to eliminate the dark current it was necessary to cover the larger part of the photo-electric cell with alkali metal, leaving only a small hole free for the entrance of light, and to use, not a single ring electrode, but a ring electrode with fine wires crossing each other and forming a wire net. By this means there is established a more or less uniform field between the sensitive surface and the anode, and only few electrons will pass through it and accumulate on the opposite glass surface where the light enters. A description of the construction of these cells has been given in the *Physical Review*, **7**, 62, 1916.

The preliminary experiments have been made with these cells; their diameter is 3.2 cm, the anode loop 2 cm in diameter, and the opening for the beam of light 1.5 cm. The cells contained alkali metals, partly sensitized and evacuated, partly sensitized and filled with argon.

In the first set of experiments a galvanometer of high sensitiveness $2.4 \cdot 10^{-10}$ was used for the measurement of the current, a constant potential was applied to the cell varying between 4 and 120 volts, the intensity of illumination was changed by moving a tungsten lamp, nearly a point-source, on a photometer bench. Some of the curves obtained are given in Fig. 1, which shows the deviations from straight lines for different conditions. It was soon observed that the curves approached a straight line when the source of light was rather weak, 6.5 candles for instance, and when the cross-section of the entering beam of light was sufficiently decreased. In the last case, even when a tungsten lamp of 388

candles was moved from 300 to 40 cm away from the sensitive surface, and with an opening of 7 mm, I found a straight line on

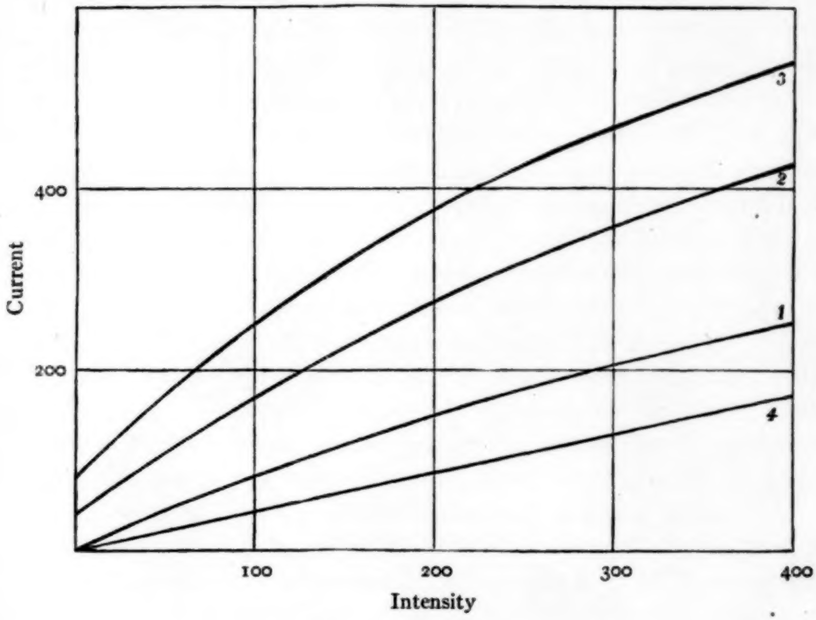


FIG. 1

plotting the data obtained; but calculating the ratio of illumination to the current I found a small variation in that ratio, as will be seen from Table IV.

TABLE IV

d	I	I/d	d	I	I/d
5.05	11.1	220	68.6	156.2	228
11.25	25.0	222	89.5	204.0	228
20.1	44.5	222	122.3	278.0	227
44.2	100.0	226	175.6	400.0	228

These data were obtained by means of an evacuated potassium cell without the treatment with hydrogen, under the influence of strong light. This result pointed strongly toward the final result of a straight line, especially because it was found later that cells partly filled with argon or helium give more satisfactory results than those

without the inert gas. The results obtained are independent of the alkali metal; sodium, potassium, and rubidium under otherwise the same conditions gave exactly the same results. In curve 4 of Fig. 1, 215 volts indicate that the potential-difference between the electrodes of the cell and the current through the tungsten lamp are kept constant. As long as the galvanometer was used, rather high light-intensities were required in order to get a sufficient current. In the next set of measurements the current was measured by means of an electrometer and a high graphite resistance, as recommended by Ives. These resistances are very satisfactory when properly treated and kept in a dry atmosphere. They can be changed readily from 10^7 to 10^{11} ohms. They are constant as long as the potential-difference applied is less than 1 volt, but when the tension increases, deviations from Ohm's law will be observed. The openings for the beam of light were 0.5 cm in diameter. Some of the results are represented by Fig. 2, which seems to consist of a series of straight lines. But it was observed in these series that the last point for the highest intensity was a little too high without exception, while one should have expected that point to be a little too low, because the potential-difference between the electrodes of the cell must have been decreased sometimes as much as 0.7 per cent, but usually less. Table V shows the data, which are represented by curves 1 and 5 of Fig. 2.

TABLE V

d	*I*	*I/d*	*d*	*I*	*I/d*
7.15	11.1	155	5.55	11.1	200
15.9	25.0	157	12.4	25.0	202
28.5	44.5	156	22.05	44.5	202
63.6	100.0	157	50.65	100.0	198
98.6	156.2	158	80.15	156.2	194
128.2	204.0	159	105.1	204.0	195
175.7	278.0	158	145.0	278.0	192
212.0	331.0	157	173.8	331.0	191
262.2	400.0	154	213.3	400.0	190

Though the curves are nearly straight, there is a slight variation in I/d in these experiments, always in the same sense, namely, that with increasing intensity I, the current d becomes too large. It will be seen in addition that as long as the intensity of illumi-

nation varies only from 11 to 100 the current remains within 1 per cent proportional to the illumination. Now, in stellar photometry, the brightness of the stars varies hardly more than 10 per cent, and the illumination remains 10,000 to 100,000 times smaller than in these experiments, and it has always been found that the current becomes more nearly proportional to the intensity as the light decreases. Hence these cells in the form prescribed are

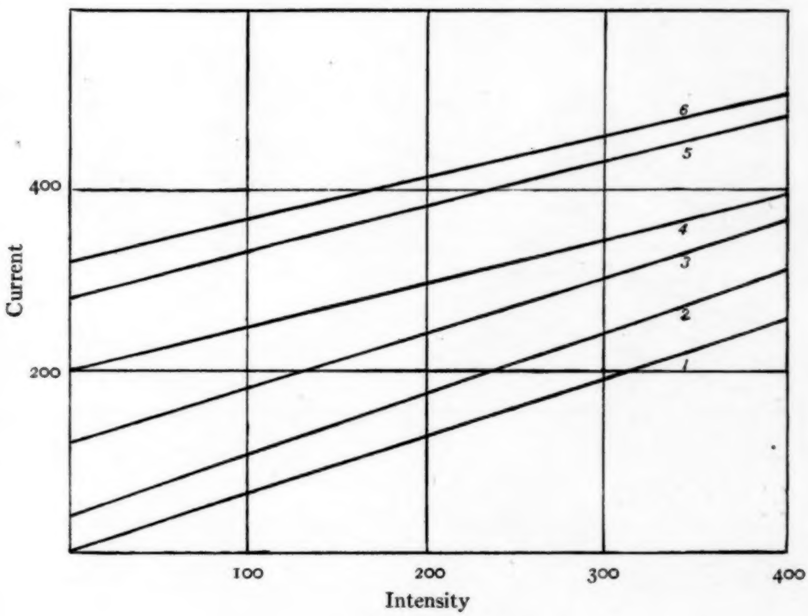


FIG. 2

sufficiently accurate for stellar photometry and for many other purposes; for instance, in plant physiology. On the other hand, if these cells are used for wider variations of the illumination a calibration is to be recommended. So, for instance, J. B. Nathanson, in the reflecting power of the alkali metal,[1] took the deviation from the straight-line relation into account. The intensity has been varied from 11.1 to 400, or from from 1 to 36. To the potassium-hydrogen-argon cell, 120 volts were applied; the tungsten lamp

[1] *Astrophysical Journal*, **44**, 137, 1916.

moving between 50 and 300 cm from the sensitive surface had a brightness of 40 candles in curves 1 and 2 of Fig. 2; 83 candles in curves 5 and 6; of 0.06 in curve 4, and of 1.02 candles in curve 3.

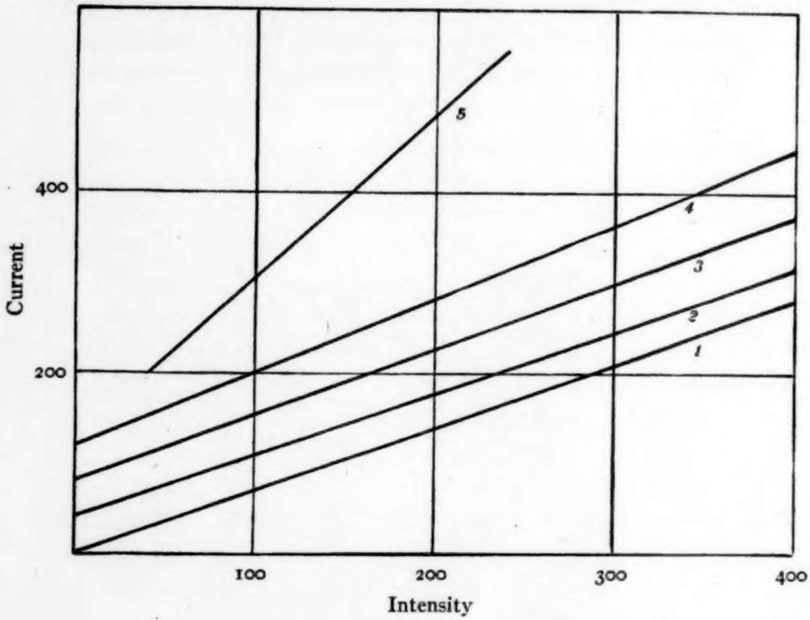


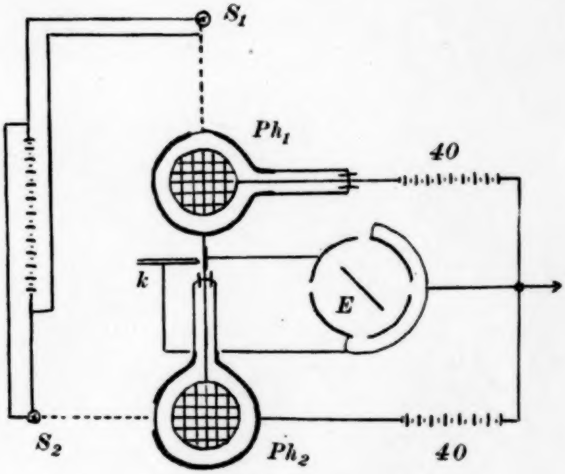
FIG. 3

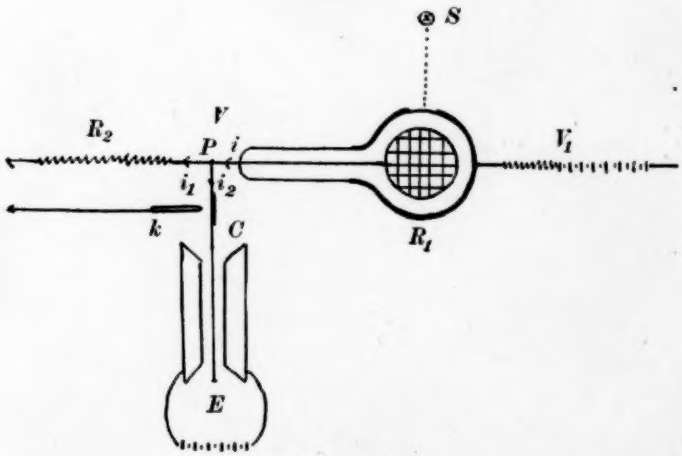
TABLE VI

d	I	I/d	d	I	I/d
9.0	12.8	142	9.63	12.8	134
34.5	48.8	141	35.2	46.8	133
70.9	101.3	142	75.2	100.0	134
106.5	151.5	142	112.8	152.0	136
139.6	200.0	144	147.7	200.0	136
186.6	268.0	144	196.6	269.0	137
251.8	360.0	144	244.0	336.0	138
276.7	397.0	144	264.0	362.0	138
....	....	....	289.2	400.0	138

The light-intensity was also varied by means of the polarization by two Nicol prisms and by means of the rotating sector disk. Some of the corrected data are given in Fig. 3 and in Table VI, curves 1 and 2 corresponding to the variation of light by means of Nicol prisms, curves 3 and 4 to the variation by revolving sector

disks. The points of curve 4 marked with crosses were obtained by the motion of the source, the points marked with circles, by means of the rotating sector disk. The two curves, respectively





FIGS. 4 AND 4*a*

straight lines, coincide exactly. This fact can be stated in the form: Talbot's law holds for the photo-electric cell. The ratio between the illumination and the current deviates again slightly from the constant value for the higher intensities, as will be seen from Table VI, which corresponds to curves 1 and 3 of Fig. 3.

Instead of a graphite resistance, an electrolytic resistance of dilute cadmium sulphate and cadmium-electrode, and, finally, a photo-electric cell itself, were used as a resistance. In the latter case, the arrangement of the experiment is given by Fig. 4. The photo-electric cell Ph_2, illuminated by the source S_2, serves as resistance; k represents a key; the source S_1 was given various distances from the photo-electric photometer. Over a short interval of intensities the required relation is a straight line, as indicated by curve

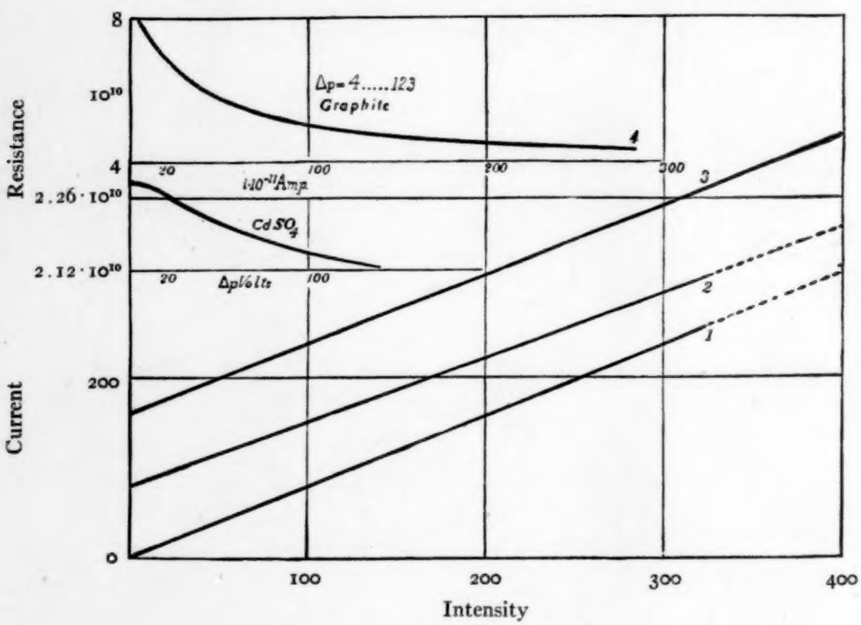


FIG. 5

5 of Fig. 3. This system is exceedingly sensitive to small variations of illuminations, but on this account it requires great care; there is probably no other photometer so sensitive as is this system.

Two other methods have been used for the measurement of the current, namely, that of the accumulated charge and that of the rate of drift of the electrometer thread. The results coincide with those obtained by means of the resistances. Some of the data are plotted in curves 1, 2, and 3 of Fig. 5. Curves 1 and 3 refer

to the accumulated charge, and 2 to the drift. In these data a slight deviation from the straight line can still be observed. The method of accumulated charge is very convenient; if the leakage is well eliminated, then the photo-electric cell has the same accumulating properties as the photographic plate. The results obtained with this method show that the quantity of electricity generated is directly proportional to the quantity of incident light; this latter is proportional to the quantity of light absorbed, and therefore the quantity of electricity evolved is proportional to the quantity of light absorbed, a photo-electric law analogous to the electrochemical law of Faraday.

For the method of the rate of drift the quadrant electrometer was replaced by a string electrometer of high sensitiveness, in order to eliminate the inertia of the needle. In Fig. 4*a* let C represent the capacity of the photo-electric cell, E the string electrometer and the connecting wires. We shall choose the following initial conditions:

$$\text{for } t=0\colon\ v=0;\ i=0$$

$$\text{at } t=t\colon\ V=V;\ i=i$$

$$i=C\frac{dV}{dt}=\frac{V_1-V}{R_1}$$

Where R_1 is the resistance of the system, essentially the resistance of the photo-electric cell, V the potential of the string in the moment t, i the current.

The solution is as follows:

$$v=V_1\left(1-e^{-\frac{t}{CR_1}}\right).$$

For small values of t, we have:

$$V=V_1\left\{1-\left(1-\frac{t}{CR_1}+\tfrac{1}{2}\frac{t^2}{C^2R_1^2}\ \cdot\ \cdot\ \cdot\ \cdot\right)\right\}.$$

$$V=V_1\frac{t}{CR_1}\ ;$$

thus the potential of the string rises proportional to the time for small values of t and large values of C and R_1. In order to get

a uniform motion of the string across the field of the microscope, one should choose large capacities and large resistances; however, if these quantities are large the sensitiveness decreases, because the current

$$i=\frac{V_1-V}{R_1}=\frac{V_1}{R_1}\left(1-\frac{t}{CR_1}\right)=\frac{V_1}{CR_1^2}(CR_1-t)$$

becomes small.

We might also attempt the following arrangement: Let us attach another high resistance R_2 (for instance, another photo-electric cell) in the point P connected to the ground. The current i from the photo-electric cell splits into two parts, i_1 and i_2, i_2 being the charging current of the string between the knife-edges. We should find the following relations:

at $t=0$, we shall assume $V=0$; $i=0$

at $t=t$ $\qquad V=V$; $i=i$

$$i=\frac{V_1-V}{R_1}\,;\quad i_1+i_2=i;\quad i_1=\frac{V}{R_2}\,;\quad i_2=\frac{dq}{dt}=C\frac{dV}{dt}$$

$$C\frac{dV}{dt}=\frac{V_1}{R_1}-V\frac{R_1+R_2}{R_1R_2}$$

$$V=\frac{V_1R_2}{R_1+R_2}\left(1-e^{-\frac{R_1+R_2}{CR_1R_2}t}\right);$$

finally we get

$$V=\frac{V_1R_2}{R_1+R_2}\,;\qquad i=\frac{V_1}{R_1+R_2}=i_1\qquad i_2=0.$$

The first arrangement, especially, is much used in stellar photometry by J. Stebbins and other astronomers.

Reviewing the results so far reported, we see that the curves obtained with the galvanometer show a tendency toward saturation; that is, the current increases more slowly than the illumination. The curves obtained with the electrometer show a slight deviation from the straight line in the opposite direction. In these measurements smaller intensities and a smaller opening for the beam of light have been used than in the previous experiments. The

former effect, occurring with higher intensities of illumination, may be explained by negative charges being accumulated on the free glass surface; by volume charges between the anode and the glass wall of the cell, and, perhaps, by reflection of electrons. The opposite effect, where the current increases a little more quickly than the illumination, is not so easily explained, provided one assumes a linear relation for the primary phenomenon. Of course, surface charges and reflection of electrons might bring about very complicated effects.

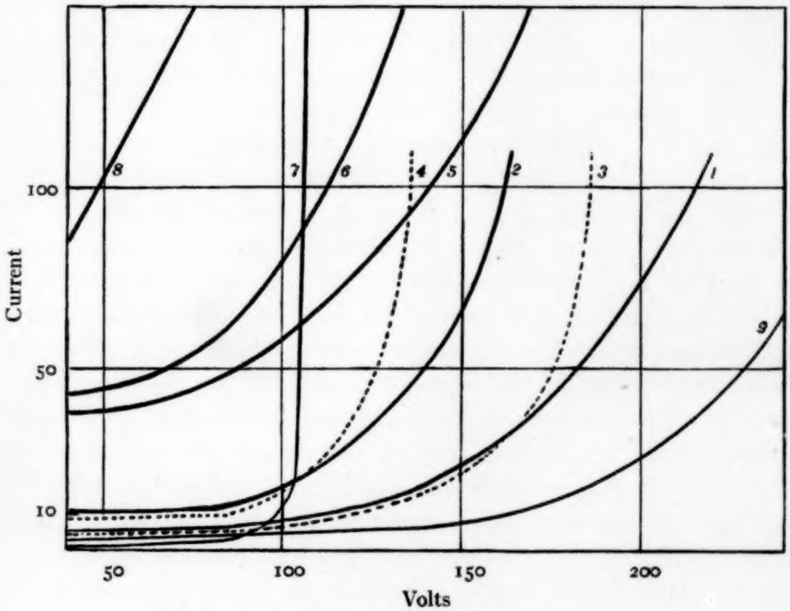


Fig. 6

Before entering upon the final arrangement of the experiment it is useful to study the characteristic curves of the cells, that is, the connection between the current and the varying potential-difference applied for a constant illumination. A very large number of such curves have been plotted. They show always the same characteristic features as will be seen from Fig. 6. The illumination is that of 2 candles at a distance of 50 cm. The curves 1 and 3 correspond to rubidium cells of the type so far used, filled at first

with hydrogen. After the formation the hydrogen was pumped out and replaced by argon. Then the curves 2 instead of 1, and 4 instead of 3, are obtained. Argon in each case raises the sensitiveness and decreases the critical value of the potential, at which the glow sets in. The curves 5 and 6 correspond to a potassium-argon cell of the same type, but of larger size, 5.8 cm in diameter. Curve 9 represents a quartz cell of the same type, in which the dark current is vanishingly small, even in comparison with the small currents which occur in stellar photometry. Curve 7 is obtained by means of the Ives cell, which gave a straight line; and curve 8 is the characteristic curve of the very sensitive cell which was used for the final measurements. In this case the curve was nearly straight.

All of these cells show practically the same features, yet they are of very different value for accurate photometric measurements. When the illumination changes over a wide interval, the potential-difference across the cell has to be kept constant. Now, it is very easy to keep the tension constant if the characteristic curve is nearly horizontal, but very hard if the slope becomes considerable. At the same time high sensitiveness is desirable in order to work with as small potential-difference as possible. For these reasons the cell of curve 5 satisfies the theoretical requirements better than the other curves. The Ives cell has a rather extreme characteristic curve. Not more than 100 volts could safely be applied. When the cells are newly formed and sealed off, the color of the sensitive layer changes more or less, and the sensitiveness falls off a little, but in the course of time the cells become constant and have been in use over six months without a noticeable change.

For the final measurements it seemed necessary to use a tube in which the electrons move in a uniform field, and where absorbed surface and volume charges are mostly eliminated. The tube is of the form indicated in Fig. 7. The cathode *K* and the anode *A* consist of aluminum disks 6 cm in diameter; the anode has a central opening for the beam of light, covered with a very fine wire net. The cathode *K* can be moved toward the anode by means of a magnet acting on a piece of iron within the tube. The cathode remains always parallel to the anode, being guided by a glass tube

which fits exactly in the outer tube. Platinum cylinders *B* protect the anode, which is connected with the electrometer, from dark currents. The alkali metal is distilled on the cathode from a side tube *S;* the whole tube is at first heated and evacuated by the aid of a mercury pump, liquid air, and charcoal.

Fig. 8 represents the final arrangement of the apparatus. Two electrometers are used: *E* for the measurement of the potential-difference Δp between the terminals of the graphite or cadmium sulphate resistance; the other electrometer E_2 only indicates the constancy of the electromotive force between the cathode and the anode of the photo-electric tube for all illuminations. When the current through the cell and the graphite resistance increases, the potential-difference applied partly appears at the terminals of the resistance and decreases, therefore, through the photo-electric cell. This change amounted to 1 volt for 120 volts, and was compensated by means of an additional storage cell and a variable resistance R_1. This compensation was carried out for each reading. When the reading was taken, the electrometer E_2 was disconnected from the rest by means of the key *k*. The current through the light source *L* was kept constant as possible. *L* was a tungsten lamp with a short straight filament. The intensity could be varied from 0.1 to 110 candles. The distance between the source and the sensitive cathode was chosen between 30 and 300 cm, giving for each experiment a variation of illumination from 1 to 100.

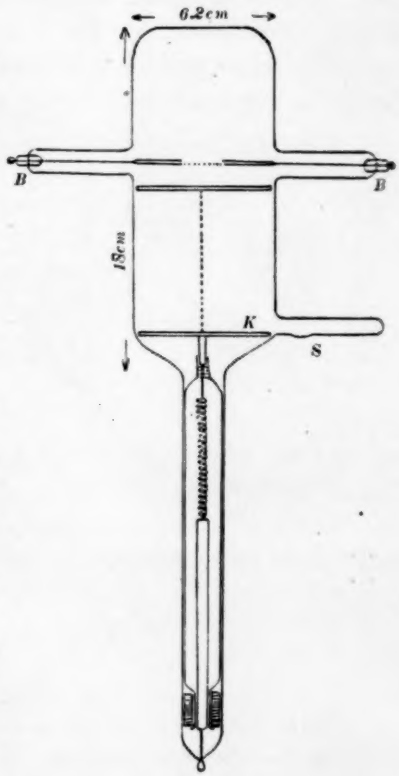


Fig. 7

The graphite resistance was occasionally replaced by the cadmium-sulphate resistance Cd. The solution was 1/1000 normal, the electrodes were cadmium. The capillary ended in two long tubes, one of which was filled with the solution while the other end was empty, hydrostatic equilibrium being established during about a day. The electrometer was repeatedly calibrated by means of a new Weston potentiometer. For each position of the lamp two readings were taken and for each series the lamp was moved from 300 cm to 30 cm from the cathode and back again to 300 cm. The

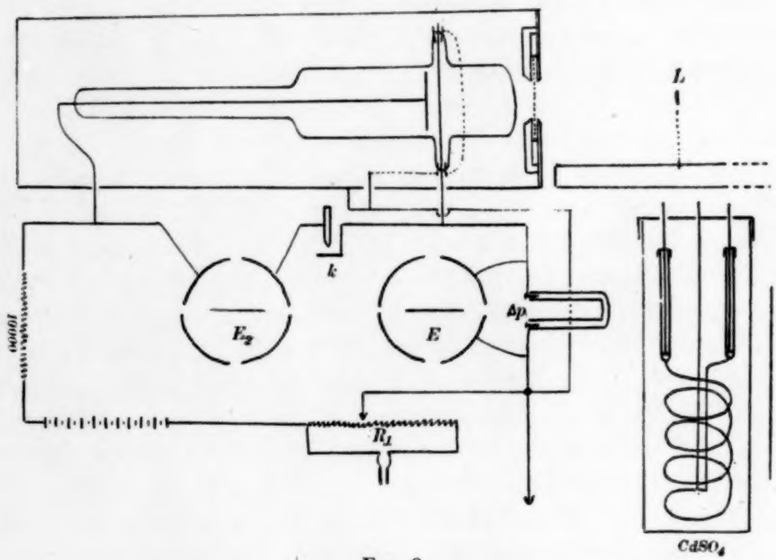


FIG. 8

readings for the same position coincided almost exactly within less than ½ per cent. Table VII gives the results of three series for a potassium cell without gas. *C* means candle-power.

The ratio between the illumination *I* and the current *d* remains constant within 1 per cent for variation of relative light-intensities from 4 to 400.

If, however, greater intensities were chosen, with increasing intensity the ratio *I*/*d* began to decrease slightly and for a source of 100 candles the deviation from a straight line was very marked, as will be seen from Table VIII.

A probable explanation of the retarded increase of the current with increasing illumination can be sought in the reflection of electrons from the anode, or in the space charge between the electrodes. If this were the case we should expect that the

TABLE VII

I	*C*=1		*C*=2		*C*=4	
	d	*I/d*	*d*	*I/d*	*d*	*I/d*
4	3.23	124	3.13	128	3.0	134
9	7.25	124	7.05	128	6.78	133
16	12.9	124	12.5	128	12.0	133
36	29.01	124	28.2	128	27.15	133
100	80.9	124	78.1	128	75.0	133
144	118.6	123	113.3	128	108.5	133
178	145.0	123	140.0	127	132.9	134
225	182.0	124	176.0	128	168.9	133
294	238.0	123	232.8	127	222.0	133
352	....	....	277.5	127	263.0	134
400	324.0	124	313.7	128	299.0	134

presence of argon in the tube would decrease the influence of reflection. We shall, in fact, see that up to the highest intensities that have been used, i.e., a source of 100 candles, there is no deviation from the straight line if the tube contains argon. For

TABLE VIII

I	*d*	*I/d*	*I*	*d*	*I/d*
4	4.07	98.5	178	141.7	126
9	9.1	99.0	225	173.6	130
16	15.55	103.0	294	224.5	131
36	33.6	107.0	352	261.7	134
100	84.7	118.0	400	294.5	136

a deflection of 140 mm we find the following distribution of the applied potential-difference: $123=\Delta p=\Delta p_1+\Delta p_2$; i=current, Δp_1=potential-difference across the cell, Δp_2=potential-difference across the graphite resistance, R_1=resistance of the photo-electric

cell, R_2 = resistance of the graphite. To a deflection of 140 mm corresponds a potential-difference $\Delta p_2 = 0.499 = 0.5$ volts.

$$R_2 = 8 \cdot 10^{10} \text{ ohms}; \quad i = \frac{p_2}{R_2} = 6.2 \cdot 10^{-12}$$

$$\Delta p_1 = 122.5; \quad R_1 = \frac{\Delta p_1}{i} = 1.98 \cdot 10^{13}; \quad \frac{R_1}{R_2} = 250.$$

These high graphite resistances are not constant above 1 volt, as is illustrated by curve 4 of Fig. 5, where the resistance decreases from 8 to $4.3 \cdot 10^{10}$ as the voltage rises from 4 to 123 volts. The cadmium sulphate resistance also decreases with increasing voltage, but not so strongly, for instance, when the e.m.f. changes from 2 to 128 volts, the resistance only decreased from 17.2 to 16.5. Below 1 volt both resistances are constant. The carbon resistance is also constant when it is of smaller order of magnitude. In some cases a carbon resistance as low as $1.44 \cdot 10^7$ ohms was used and then it was perfectly constant. It is therefore advisable to use as small a resistance as possible and an electrometer of high sensitiveness.

Tables IX, X, and XI contain the measurements made with the previous tube, with parallel electrodes, filled with argon.

$C = 0.6$; $CdSO_4$ resistance

The last series was taken with a galvanometer. Within 1 per cent the ratio between the illumination and the photo-electric current is constant for an interval of intensities from 1 to 100, and for light-sources from 0.16 to 100 candle-powers. For a tungsten lamp of 100 candles there may be a slight increase of the ratio I/d for the highest intensities, or 1110 candle-meters. The smallest intensity in this series was 0.018 candle-meter. As already mentioned, readings were taken as the lamp was moving toward and away from the cathode of the cell; often a small decrease of the deflection in a given position was observed, when the source of light was receding from the photo-electric cell.

On the basis of this result we can establish a logical connection between the photo-electric, the electro-chemical, and the photochemical effects. Let us consider the following arrangement. The anode of a photo-electric cell is earthed. The cathode is

connected with the anode of a voltameter, for instance a silver nitrate voltameter, the cathode of which is grounded. If light falls on the cell, the current will deposit silver according to Faraday's

TABLE IX

I	d	I/d	I	d	I/d
4	10.02	399	36	90.3	399
9	22.6	399	100	250.6	399
16	40.1	399	....	....	....

TABLE X

I	$C=0.16$		$C=3.5$		$C=28$	
	d	I/d	d	I/d	d	I/d
4.........	3.1	129	2.7	148	2.8	143
9.........	6.92	130	6.04	149	6.29	143
16.........	12.2	131	10.7	150	11.25	142
36.........	27.4	131	24.2	149	25.3	142
100.........	76.8	130	66.9	150	70.5	142
144.........	110.0	131	97.3	148	101.0	143
178.........	136.4	131	119.5	149	125.7	142
225.........	171.6	130	152.0	148	158.5	142
294.........	227.8	129	199.4	148	206.0	142
352.........	271.0	130	237.8	148	245.1	143
400.........	310.6	129	270.9	148	278.9	143

TABLE XI

I	$C=100$		$C=34$	
	d	I/d	d	I/d
4........	3.07	130	2.31	174
9........	6.92	130	5.12	175
16........	12.3	130	9.15	175
36........	27.7	130	20.7	174
100........	77.1	130	57.7	174
144........	111.0	130	83.0	174
178........	137.0	130	102.8	174
225........	173.0	130	129.9	174
294........	224.0	131	169.8	174
352........	267.0	132	203.0	174
400........	304.0	132	230.5	174

law. The quantity of silver deposited in a given interval of time will be proportional to the quantity of light absorbed by the photo-electric cell. But this statement is a special case of the

fundamental law of photo-chemical action. If, therefore, light produces a chemical effect directly, we have, as it were, a combination of the photo-electric and the electro-chemical processes. But whether ionization and electric conduction can be detected in each photo-chemical reaction seems to be an open question. If in the photo-electric process $it = CIt$, and if in the electro-chemical process $M_{ag} = a_{ag} it$, then it follows for the photo-chemical reaction

$$M_{ag} = a_{ag} CIt.$$

SUMMARY

The photo-electric current is not proportional to the intensity of illumination in the older spherical form of the cell. If, however, the cell is mostly covered with a metallic layer and the anode has the form of a circular wire net, the deviation from the straight line is very small so that this cell can be used for many photometric purposes, for instance, in stellar photometry, in plant physiology, for the measurements of reflecting power and of coefficients of absorption and transmission; the Ives cell, at least for small illumination, gives a straight line. In the cell used in the final measurements, two electrodes are parallel, the field is uniform, and the current is proportional to the illumination over a very wide range if the cell contains argon, over a smaller range if the cell is evacuated. Talbot's law holds for the photo-electric phenomenon. A simple relation between the photo-electric, the electro-chemical, and the photo-chemical processes has been pointed out.

This study is being continued in several directions. The very strong, the very weak, illumination, and the action of ultra-violet light require further investigations. The Faraday cylinder method also will be used.

I am very glad to acknowledge my indebtedness to Director E. P. Hyde of the Nela Research Laboratory, where this investigation was carried out, and to his staff, for all the facilities placed at my disposal, for their helpful suggestions, and for the very good time spent in an excellent company of human scientists.

University of Illinois
Laboratory of Physics
January 1917

HIND'S VARIABLE NEBULA N.G.C. 1555[1]

By FRANCIS G. PEASE

Hind's variable nebula N.G.C. 1555[2] lies close to the irregular variable star T. Tauri. Since December 1911 seven negatives of this object have been made from time to time with the 60-inch reflector. The length of the exposures, which were on Seed 27 plates, has usually been about three hours.

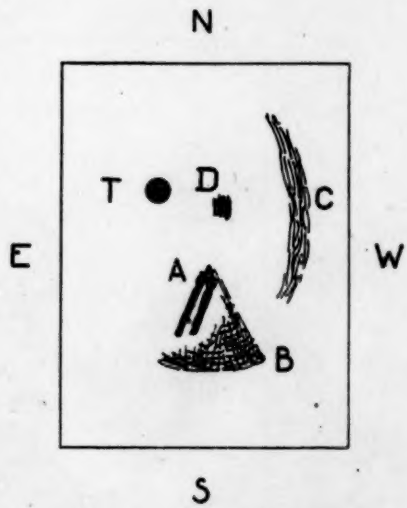


FIG. 1.—T Tauri and Hind's variable nebula.

The most prominent feature is a fan-shaped bit of nebulosity, whose apex lies 220°, 25″ southwest of T Tauri and points a little to the west of the variable (Fig. 1). Two knots at *A*, each with a streamer running to the south, are the brightest parts of the nebula. The sides of the fan include an angle of 70° and are about one minute of arc in length. The nebulosity at *B* is never strong, but on some plates shows distinct detail. A curved streamer lying to the west of *T* is also relatively faint and at times only portions of it show. A knot appears at *D* and varies in size and intensity. A nebulous wing of irregular intensity extends from the variable star about 7″ to the south. There is evidence of very faint extended nebulosity filling the whole starless region about T Tauri. Such details as can be seen on the photographs are roughly sketched in

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 127.

[2] Position for 1860: $\alpha = 4^h\ 13^m\ 48^s$, $\delta = +19°\ 11'.2$. N.G.C. 1554, which precedes No. 1555 by 15^s and is $0'.2$ south, is sometimes referred to as Hind's variable nebula, probably because of the note on p. 225 of the *First Index Catalogue*.

Fig. 2. The luminosity at *E*, *F*, *G*, and *H* is strongest on plate No. 55, although it is also visible in varying degrees on the other plates of the series. The regions I and K appear dark on the plates.

Below is a brief description of the appearance of the nebula on each plate. The brightness of T Tauri is about the same on all the photographs excepting Nos. 180 and 181, upon which it is about a magnitude fainter. Although both the star and the nebula

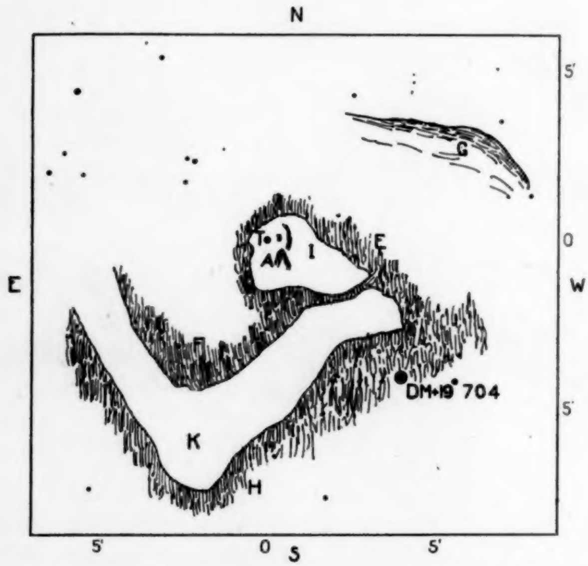


FIG. 2.—Faint nebulosity surrounding T Tauri and Hind's variable nebula

increased in brightness during the interval separating Plates 181 and 217, the data now available do not establish a parallelism between the fluctuation of the two objects.

Plate No. 55, 1911, December 25, exposure $3^h\ 39^m$. Nebula diffuse in outline.

T strong.

A knots strong; wings to south medium.

B and *C* very faint.

D medium intensity.

Plate No. 180, 1913, January 8, exposure 20^m.

T faint; has wing on south side, no other detail visible.

Plate No. 181, 1913, February 3, exposure $2^h\ 30^m$. Nebula fan-shaped; contrast stronger than No. 55.

T faint; has strong wing on south side.

A knots stronger than in No. 55; wings to south medium.

B very faint.

C and *D* missing.

Plate No. 217, 1913, September 28, exposure 3^h. Nebula fan-shaped; sharp detail and strong contrast; the nebula is appreciably stronger in intensity than on No. 181.

T same as No. 55.

A knots about same as in No. 181; wings to south medium and longer.

B shows detail; medium intensity.

C faint.

D trace.

Plate No. 226, 1913, October 27, exposure $3^h\ 30^m$. Nebula fan-shaped; detail softer.

T strong; shows wing on south side.

A knots blend with wings; northern half strong.

B west edge only visible; medium intensity.

C very faint.

D faint.

Plate No. 230, 1913, November 25, exposure 3^h. Nebula fan-shaped; same as No. 226.

T strong; wing on south side stronger than in No. 226.

A weaker than No. 226; point strongest; left wing strong, right weak; cross-bar halfway down.

B details; medium intensity.

C missing.

D trace.

Plate No. 280, 1916, November 27, exposure 3^h. Whole region embracing *A*, *T*, and *C* filled with diffuse nebulosity.

T strong.

A diffuse knot, appreciably weaker than on No. 226.

Mr. Adams finds that the spectrum of T Tauri is of type Md with additional bright lines, and that the parallax is of the order of

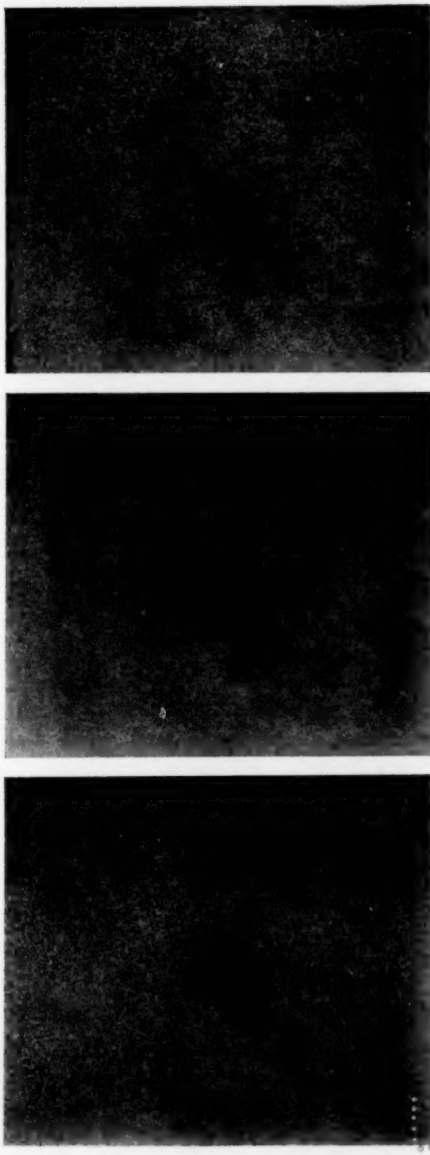
$0\overset{''}{.}05$ to $0\overset{''}{.}10$. Since the bright lines of the spectrum appear to extend beyond the dark lines, they are evidently due to the surrounding nebulosity.

In Plate II are shown enlargements of photographs Nos. 181, 217, and 280. The enlargement and contrast necessary for purposes of reproduction have emphasized the grain of the photographs, but the changes in form and intensity of the nebula are clearly visible.

Mount Wilson Solar Observatory
January 1917

PLATE II

N



No. 181 E
1913, Feb. 3

W

No. 217
1913, Sept. 28

No. 280
1916, Nov. 27

HIND'S VARIABLE NEBULA N.G.C. 1555

Exposures about 3^h with the 60-inch reflector
Enlargement 4.5, Scale 1 mm = 6."1

SOME SPECTRA IN THE PHOTOGRAPHIC INFRA-RED

By CHARLES F. MEYER

In the summer of 1912 the author, while working at Bonn, in Professor Kayser's laboratory, photographed the spectra of a number of elements in the infra-red. The photographs were made at the suggestion of Dr. Keivin Burns.

The spectroscopy of the infra-red has not been worked up with anything like the degree of completeness which is desirable. It is important that all spectra should be investigated photographically to as great a wave-length as possible, for both the degree of accuracy which can be attained and the number of lines which can be observed are greater than by other methods. Perhaps the most complete work in this field has been done by Lehmann,[1] but his wave-lengths are well known to be very inaccurate and this makes his tables almost valueless. Many others have photographed in the infra-red, but the total amount of work which has been done is small compared to that which should be done. As a rule the wave-lengths do not go as far out as they might go, and those of many lines, even of many prominent ones, differ by several angstroms as determined by different investigators. Since the photographs made by the author were obtained, some work has been published by Lorenser,[2] and some by Eder,[3] and in the present paper these investigations will serve especially for comparison.

The spectra photographed were those of salts of lithium, potassium, calcium, barium, and strontium, in the carbon arc. For the final exposures, chlorides were always used, though other salts were occasionally tried. Some photographs of the mercury arc spectrum were also taken, but they yielded nothing worthy of comment.

[1] *Annalen der Physik*, **5**, 633, 1901; **8**, 643, 1902.

[2] *Inaugural Dissertation*, Tübingen, 1913.

[3] *Sitzungsberichte d. k. Akad. der Wiss.*, Wien, **123**, 1914.

A Rowland concave grating of one meter focus, and 16,000 lines to the inch, was used to analyze the spectra. The slit was placed in such a position that the region 6400 to 9600 A, of the first order, fell into the camera. The second-order iron spectrum served as comparison. While the exposure for the infra-red lines was being made, a red glass screen was placed in front of the slit, in order to eliminate the second-order lines of the spectrum which was being examined. This screen proved to be absolutely opaque to the second order, even for very long exposures. A transmission-curve of the glass, kindly made for me by Dr. Pfund, showed it to be opaque to 5800 A. At this point transmission begins, and rises gradually to about 50 per cent at 9000 A. The camera-box was made for the use of films, and accordingly these were used. They were dyed by a method which was a modification of Wallace's by the addition of dicyanin.

The width of the slit used was about a tenth of a millimeter, and the length of the exposure varied from 15 minutes to 3 hours. The great width of the slit and length of exposure interfered materially with the definition, and, consequently, with the accuracy which could be attained in measuring. The spectra were not photographed with a view to obtaining the greatest possible accuracy of wave-length, and it was decided to be content with fair accuracy. The values given by Lorenser, and by Eder, are, on the whole, better than those in this work, though there is good reason to doubt whether Eder's wave-lengths are throughout as accurate as might be supposed.

The measures were made at the Detroit Observatory, where Professor Hussey was so good as to place a measuring engine at the author's disposal. The measurement of the spectra has been delayed, as the author did not have access to an engine and other equipment for some time after making the photographs.

The wave-lengths are on the international system, being based upon Burns's table of the iron spectrum. Wave-lengths in the Rowland system are about 0.3 A higher than those on the international, in this region of the spectrum, and this should be borne in mind in making comparisons in the tables which follow.

The intensities are given on a scale of 1 to 10. Where the intensities of lines observed only by others are given, they have been roughly adapted to this scale.

All films were examined for the presence of the so-called Lyman ghosts.[1] By determining the positions of the ghosts of certain strong lines in the green and red, the positions which would be occupied by the ghosts of all strong lines could be determined, and it is felt that all such ghosts have been identified and eliminated.

LITHIUM AND POTASSIUM

(Impurities: Na, K, Ca, Rb, in the Li spectrum;
Na, Li, Ca, Ba, Rb, in the K spectrum)

The spectra of lithium and potassium showed no new lines or bands. In the lithium spectrum the lines 6708 and 8127 come out strongly, and impurities show to 8540, so that even fairly faint lines should show between 8000 and 9000 if they existed. The wave-length of the 6708 line is accurately known, and the line was not measured. The wave-lengths obtained by the author and others[2] for the 8127 line are given in Table I.

TABLE I

LITHIUM

International System		Int.	Remarks	Rowland's System		
Meyer	Eder			Paschen	Lewis	Saunders
8126.6	8126.27	10	Broad	8127.1	8126.8	8127.0

Eder's is the only previous value based on international standards. My value falls more nearly into line with the older values, based on Rowland's standards, when the correction of 0.3 A is applied.

The potassium spectrum in this region consists of two strong pairs, the longer pair reversed, and, in addition, one rather faint

[1] *Proceedings of the American Academy of Arts and Sciences*, **36**, 1901; **39**, 1903.

[2] All wave-lengths given for comparison will be found in Kayser's *Handbuch der Spectroscopie*, **5**, **6**; except those due to Lorenser and Eder, which are given in the papers already referred to.

line. Impurities show on my films only to 8200, but are well seen out to this point. We cannot feel quite confident in this case that any lines below 9000 would have been found. The wave-lengths are given in Table II.

TABLE II

POTASSIUM

International System		Int.	Remarks	Rowland's System	
Meyer	Eder			Kayser and Runge	Saunders
7698.9	7699.02	9	Reversed	7699.3	7699.08
7664.8	7664.95	10	"	7665.6	7664.91
6965.3	....	2	....	....	6966.3
6939.4	6939.07	6	Broad	6938.8	6939.5
6911.9	6911.31	5	"	6911.2	6911.8

My values for the second pair seem to be too high. The faint line has been missed by most investigators; possibly it is an impurity, but there is a line called for in about this position in the second subordinate series.

CALCIUM

(Impurities: Na ?, K)

Calcium chloride produces in the spectrum, besides the metallic calcium lines, also bands which are characteristic of the salt itself. The table of metallic lines follows (Table III). The first three lines are very faint. All efforts to explain them as spurious failed, however, so they have been retained.

Lorenser tabulates, from 7984 to 7468, eleven lines. He used in the arc at times metallic calcium and at times calcium chloride. These lines were not found either by Eder, using metallic calcium, or by me, using chloride. Why Eder did not observe the lines is not entirely clear, as their intensities range from 1 to 6 on the foregoing scale. On my films, part of the region is obscured by a band, and the remainder is not entirely free from continuous background. The strongest of these eleven lines (7610.66) is present on two of my films, but with nothing like the relative intensity assigned by Lorenser, and I had attributed it to barium, though probably falsely. There is also a faint suggestion of the

line which is next, both in intensity and position, namely 7602.78. Lorenser noted that a group of five of these eleven lines, including the two just mentioned, did not appear in the vacuum arc. The group is characterized by being unsymmetrical to the red.

TABLE III

CALCIUM

International System		Int.	Rowland's System
Meyer	Eder		Lorenser
9166.2	....	1	....
9122.0	....	1	....
9103.0	....	1	....
8662.0	....	3	8662.50
8541.8	8542.25	5	8542.47
8497.8	8498.11	3	8498.35
8153.1	....	2	8153.13
......	....	....	?8077.8
7995.0	....	2	7995.31
			7984 to 7468 Eleven lines
7326.0	7326.10	9	7326.43
7202.1	7202.15	8	7202.51
7147.9	7148.14	10	7148.49

Three chloride bands appear on my films. They show very distinctly, and a number of the component lines are sharp enough to measure. The bands run to longer wave-lengths, and shade off in that direction also. The first and last are simple, that is, they have but one head. The one of intermediate wave-length is more difficult of interpretation. There are probably two heads, one at 8166 and one at 8153, about coincident with the metallic line given in the table above. So far as I know, bands have not been photographed in this region of the spectrum before. The existence of these bands was noted by Becquerel, using the phosphor-photographic method.

BARIUM

(Impurities: K, Ca)

The barium spectrum has been measured by Lehmann, Hermann, and Lorenser, basing their wave-lengths on the Rowland system. Lehmann's values are poor. Burns, Werner, George,

and recently Eder, have measured the spectrum in this region on the international system, but their tables are less complete. My films show evidence of a band at about 8000, but neither the lines nor the head of the band are distinct.

TABLE IV

Calcium Chloride Bands

8855.7	8354.4	7942.4
8849.3	8349.8	7937.5
8843.3	8344.7	....
8838.3	8338.8	7908.0
8833.1	....	7903.2
....	8329.8	7899.0
....	8325.4	7894.6
8773.3	8320.8	....
8769.2	8316.1	7885.8
8763.9	8311.4	7881.7
8759.7	8307.1	....
8755.4	8302.8	7873.8
8751.7	8298.7	....
8747.9	8294.5	....
....	8290.6	7850.7
8739.1	8286.4	7846.1
8735.6	8282.4	....
8732.4	8278.1	7822.0
8715.7	8273.4	7818.5
8713.0	8268.2	7814.8
8710.1	....	....
8706.7	8257.5	7807.6
8702.6	8253.9	....
8698.6	8248.9	7770.2
....	8245.7	7766.1
....	8242.1	7762.1
8651.0*	....	....
	....	7720.3
	8231.9	....
	8228.5	7716.8*
	....	
	....	
	8166. *	
	8153. *	

* Head.

STRONTIUM

(Impurities: K, Ca)

The strontium spectrum has no lines in the photographic infra-red, except for two at the visible limit. Eder claims to have found two new lines, but these are undoubtedly the sodium pair,

TABLE V

International System		Int.	Rowland's System	
Meyer	Eder		Hermann	Lorenser
8914. *	....	2	8915.19	8915.40
8860.5	....	1	8861.40	8861.32
8799. *	....	1	8799.86	....
....	....	1	....	8659.65
8653.6	....	2	8654.33	8654.40
8581.6	....	2	....	8582.66
....	....	1	....	8570.34
....	....	1	....	8569.60
8567.1	....	1	....	8568.08
8559.7	8559.98	5	8560.20	8560.21
....	....	1	....	8542.72
8521.9	....	1	....	....
8513.7	....	1	....	8514.50
8414.2	....	2	....	....
8350.5	....	1	....	....
....	....	3	....	8329.17
8325.2	....	2	....	....
....	....	1	....	8288.41
8284.8	....	2	....	....
8262.8	....	1	....	8264.30
8254.3	....	1	....	....
8224.0	....	1	....	....
8210.2	8210.33	4	8210.73	8210.63
8161.9	....	1	....	8162.09
8158.1	....	1	....	8158.56
8147.6	....	2	8148.32	8148.14
8120.5	....	2	8120.84	8120.88
8018.0	....	1	....	8018.64
7982.4	....	2	....	7982.75
7961.4	....	1	7961.23	7961.47
7957.6	....	1	....	7957.61
7939.3	....	1	7939.21	7939.79
7911.5	7911.35	7	7911.53	7911.67
7905.8	7905.77	7	7906.13	7906.12
7878.2	....	2	7878.13	7878.34
....	....	1	....	7865.51
....	....	1	....	7863.74
7839.8	7839.56	3	7839.57	7839.82
....	....	2	....	7829.25
7798.5	....	1	....	7798.65
....	....	1	....	?7783.84
7780.5	7780.55	7	7780.70	7780.77
....	....	3	....	7775.74
....	....	1	....	?7770.55
7766.8	7766.81	1	....	7767.19
7751.6	7751.74	5	7751.92	7752.02
7721.8	....	3	....	7722.13
....	....	....	7709.96	....
7706.6	7706.59	5	7706.82	7706.88
7672.0	7672.10	10	7672.42	7672.48
7668.2	....	1	....	....
....	....	3	....	7662.31

*Estimated

TABLE V—*Continued*

International System		Int.	Rowland's System	
Meyer	Eder		Hermann	Lorenser
7642.9	7642.88	6	7643.42	7643.31
7636.9	7636.89	6	7637.47	7637.29
....	....	1	....	7616.83
7610.5	7610.46	6	7610.74	7610.83
....	....	2	....	7575.49
7543.2	....	1	....	7543.88
7527.8	....	1	....	7528.60
7523.1	....	1	....	7523.93

present as an impurity (see below). If we subtract 0.22 and 0.23 respectively from the values given by Eder for the strontium pair (8195.14 and 8183.58), we get his values for the sodium pair, and, moreover, the character and relative intensities of the lines are the same in both cases. My films show impurities to 8660, but they show no trace of these lines, and Lorenser did not find them, either.

TABLE VI

Strontium

International System		Int.	Remarks	Rowland's System
Meyer	Eder			Lorenser
....	8195.14	3	Impurity (Na)	....
....	8183.58	3	"	....
7673.0	7673.07	3	....	7673.38
7621.5	7621.55	3	....	7621.76

There are some chloride bands present in the infra-red. The one of longest wave-length has a fairly well-marked head at 8258, but the individual lines are not distinct enough to measure. They run to longer wave-lengths and shade off in that direction also. The next is a band group, apparently. Lines are distinguishable by the aid of a lens between 8200 and 7900, but only a few are distinct enough to measure. The lines which were measured, and the heads, are given in Table VII.

SODIUM AND RUBIDIUM AS IMPURITIES

The wave-lengths of the infra-red sodium pair were in doubt by several angstroms until they were recently measured by Eder as reversed lines. I did not photograph the sodium spectrum as such,

TABLE VII

STRONTIUM CHLORIDE BANDS

....	Lines not distinguishable
......	
8258	Head (measured)
8044.2....................	Line
8040.3....................	"
8036.5....................	"
8032.5....................	"
8028.7....................	"
8024.6....................	"
......	
......	
7902	Head (estimated)
7883	" (measured)
7874	" (estimated)
7866	" "
7860	" "

but this pair appeared quite plainly in my lithium and potassium spectra, but not reversed. The recent values, and the better of the older ones, are given in Table VIII.

TABLE VIII

SODIUM

International System		Int.	Rowland's System	
Meyer	Eder		Saunders	Hermann
8195.1	8194.92	10	8196.1	8195.33
8183.4	8183.35	7	8184.5	8183.74

The two strong rubidium lines also appeared in my lithium and potassium spectra, and were measured (Table IX).

In conclusion, I wish to thank Dr. Burns for suggesting this work and for the help he has given me during its progress. I also

TABLE IX

RUBIDIUM

International System		Int.	Rowland's System	
Meyer	Eder		Eder and Valenta	Saunders
7947.5	7947.63	5	7947.7	7947.6
7800.0	7800.30	10	7800.3	7800.2

wish to thank Professors Kayser, Hussey, and Curtiss for the kindness they have shown in placing facilities at my disposal.

PHYSICAL LABORATORY
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DEPENDENCE OF THE SOLAR APEX UPON PROPER MOTION AND CAUSE OF THE DIFFERENT POSITIONS OF THE APEX YIELDED BY RADIAL VELOCITIES AND PROPER MOTIONS

By C. D. PERRINE

In continuation of the investigations detailed in *Astrophysical Journal*, **43**, 286, 1916, which showed an apparent dependence of the position of the solar apex upon the sizes and parallactic signs of the proper motions of the stars used in the determinations (from radial velocities), apices have now been derived from proper motions of the same stars. The results from proper motions confirm satisfactorily the dependence found from the radial velocities.

The results both from radial velocities and from proper motions are given in Table I. On account of the small number of stars in the divisions of large and medium-sized proper motions of the classes A, F, and G and the uncertainty of positions determined from so few proper motions, these three classes were combined. It was necessary to reject a very few of the largest proper motions, when they came in such small groups that to have included them would have been to produce a very abnormal effect upon the result obtained. In the present case only the most extreme cases were rejected—9 out of a total of 341 stars in the three groups where rejections were made. Even large values were not rejected if there were sufficient stars in the group reasonably to reduce the effect. Such a course is unavoidable where data are limited, and is justified, in my opinion, upon the ground that the result is more representative than if such observations had been included. An examination of the solutions shows that the conclusions arrived at are in no way affected by these rejections.

The solutions were made by the method of least squares, with the use of the following formulae:

$$-X \sin \alpha + Y \cos \alpha = \mu\alpha$$

$$-X \cos \alpha \sin \delta - Y \sin \alpha \sin \delta + Z \cos \delta = \mu\delta$$

$$\tan A = \frac{Y}{X}$$

$$\tan D = \frac{Z}{(X^2+Y^2)^{\frac{1}{2}}}.$$

Right ascensions and declinations were combined. The solutions were made as in previous work of this kind, i.e., by regions of 2^h in right ascension by 30° in declination. Probable errors were not determined. The comparatively small number of observations in many of the solutions and the nature of the data seemed not to warrant it in the present preliminary work. The reality of the phenomena must rest upon other considerations.

TABLE I

APEX FROM RADIAL VELOCITIES AND FROM PROPER MOTIONS

Class	Radial Velocities					Proper Motions		P.M.−R.V.
	A	*D*	$V_\odot$	Stars	Regions	*A*	*D*	*D*
			km					
2.9 and brighter	258°0	+41°5	−18.9	110	33	258°8	+24°6	−16°9
B	276.0	+29.6	−20.3	193	31	272.8	+32.4	+ 2.8
A, F, G								
Large	268.1	+ 8.6	−26.5	141	40	261.3	+22.1	+13.5
Medium	269.0	+ 7.0	−23.7	81	29	238.5	+50.8	+43.8
Small	254.7	+35.0	−17.8	277	47	263.5	+47.4	+12.4
K								
Large	288.7	+18.8	−27.9	85	31	255.9	+13.0	− 5.8
Medium	250.4	+15.1	−16.7	85	33	258.5	+42.9	+27.8
Small	260.6	+36.8	−21.3	220	45	276.4	+69.2	+32.4
Mean	265.7	+24.0	....	....	....	260.7	+37.8	+13.8
Adams' 500 Radial Velocities:								
Large	276.9	+ 3.1	−11.7	47	18	279.2	+38.4	+35.3
Medium	244.2	+41.6	−21.9	32	12	249.6	+50.3	+ 8.7
Small	272.0	+31.6	−15.9	349	41	266.4	+67.7	+36.1

The stars of W. W. Campbell's catalogue[1] were separated into two classes according to magnitude—those of 2.9 and brighter and those of 3.0 and fainter. Owing to the few stars of magnitude 2.9 and brighter, all spectral classes of these stars were combined. This is objectionable, but in the present state of our knowledge it seemed desirable to separate the very brightest stars.

[1] *Lick Observatory Bulletins*, **6**, 108, 1911; **7**, 20, 51, 113, 1913; *Publications of Lick Observatory*, **9**, 329, 1911.

Solutions were also made from the 500 stars of W. S. Adams' list[1] of radial velocities, the results of which are included in Table I. These stars also show the dependence of position of the apex upon size of proper motion.

It should be remembered in this connection, that Adams' list contains no stars south of $-26°$ and only 94 south of the equator. Forty-seven stars are in Campbell's catalogues and were, therefore, omitted in the solutions from Adams' data.

The stars on Adams' list are all relatively faint, most of them being between magnitudes 5.5 and 6.5.

The original object of these investigations was to find the cause of the discordance of about 10° which Campbell[2] noted between the declination of the solar apex as derived from his radial velocities and the position obtained by Lewis Boss from proper motions, a discordance which is confirmed by the radial-velocity apex of Hough and Halm[3] and by many other investigations of proper motions. This discordance is discussed to some extent by Eddington.[4]

The finding of a relation to the size of $\mu\alpha$ was unexpected. The results of the present investigation bear upon this discordance in the apex of the solar motion, as well as upon the phenomena of dependence upon size of proper motion, and both will be considered together.

RELATION BETWEEN POSITION OF SOLAR APEX AND PROPER MOTION IN RIGHT ASCENSION

The classifications of the A, F, and G stars, the K stars, and Adams' list show, without a single exception, much smaller declinations for the apex from large than from small $\mu\alpha$, and no greater discordances in the intermediate values than might be expected from the data. This conclusion is equally true of the results from proper motion. This discordance of position appears to be chiefly in declination. The data do not appear at present to be sufficient to justify a conclusion as to difference in right ascension, further than to say that there are smaller variations of right ascension which appear to be systematic and that so large a discordance as

[1] *Mt. Wilson Contr.*, No. 105.

[2] *Lick Observatory Bulletin*, **6**, 128, 1911.

[3] *Monthly Notices*, **70**, 85, 1909.

[4] *Stellar Movements*, p. 74.

that in D is not likely to be wholly in a single (apparently) arbitrary co-ordinate. Until more is known as to the nature of this dis-

TABLE II

COMPARISON OF APEX FROM RADIAL VELOCITIES (CAMPBELL) AND PROPER MOTIONS (LEWIS BOSS)

Spectral Class	Radial Velocities				Proper Motions				P.M.−R.V.	
	A	D	$V_{\odot}$	Number of Stars	A	D	μ	Number of Stars	ΔA	ΔD
			km							
B	276°.0	+29°.6	−20.3	193	274°.4	+34°.9	2″.40	490	−1°.6	+ 5°.3
A	260.9	+15.3	−18.3	185	270.0	+28.3	4.56	1647	+9.1	+13.0
F	267.9	+11.1	−18.9	184	265.9	+28.7	7.71	656	−2.0	+17.6
G	257.4	+20.2	−16.4	131	259.3	+42.3	5.24	444	+1.9	+22.1
K	274.2	+25.6	−20.7	391	275.4	+40.3	5.74	1227	+1.2	+14.7
M	269.7	+31.7	−22.9	65	273.6	+38.8	4.99	222	+3.9	+ 7.1
All classes: 2.9 and brighter	258.0	+41.5	−18.9	110	.....	.....	.....	.....	.....	.....
Mean	.....	+25.0	.....	.....	.....	+35.6	.....	.....	.....	+13.3

cordance and related phenomena it does not seem advisable to attempt quantitative determinations.

DIFFERENCES BETWEEN THE POSITIONS OF THE SOLAR APEX DERIVED FROM RADIAL VELOCITIES AND FROM PROPER MOTIONS, AND THEIR CAUSE

There are also clearly shown in the results in Table I (stars of magnitude 3.0 and fainter), differences between the declinations of the apex derived from proper motions and from radial velocities, those from proper motions being to the north of the apices from radial velocities. There is only one exception in ten groups. That one is the large $\mu\alpha$ group of the K stars, the contradiction being slight and the nature of the data such as to render their evidence of small weight in the face of all the other. It is conceivable that to some extent at least this difference of apex may be greatest among the stars of small proper motion.

There is some indication that the size of the discordance increases with spectral type, at least among the stars of small $\mu\alpha$. The evidence from Campbell's stars is very consistent and Adams'

list is not necessarily contradictory. This matter is, however, confused with the question of differences between the northern and southern stars and increase with decreasing brightness.

Of great interest in this connection is a comparison[1] of these positions of the apices derived from the radial velocities of 1149 stars of magnitude 3.0 and fainter of Campbell's lists by spectral class with the apices from proper motions, also by spectral class, of 4686 stars by Lewis Boss,[2] which is given in Table II. We here find not only more northerly declinations for the apices from all six spectral classes, but a systematic curve in these varying in a general way like the A, D, $V_{\odot}$, and μ from both radial velocities and proper motions.

TABLE III

PROPER MOTIONS. SEPARATE SOLUTIONS FOR NORTHERN AND FOR SOUTHERN STARS

SPECTRAL CLASS	NORTHERN					SOUTHERN					ΔD
	A	D	X	Y	Z	A	D	X	Y	Z	N.–S.
All classes: 2.9 and brighter	260°0	+26°0	+27.3	+155.3	− 77.1	255°4	+21°6	+ 26.6	+101.8	− 41.6	+ 4°4
B	281.5	+46.4	− 3.3	+ 16.3	− 17.4	269.4	+26.2	+ 0.4	+ 34.5	− 17.0	+20.2
A, F, G											
Large	258.0	+35.6	+48.	+228.	−167.	263.7	+11.2	+ 39.	+356.	− 71.	+24.4
Medium	256.3	+36.8	+ 6.3	+ 25.8	− 19.9	227.9	(+56.6)	+ 29.8	+ 32.9	− 67.3	(−19.8)
Small	262.2	+59.4	+ 2.4	+ 17.2	− 29.4	264.5	+40.1	+ 3.2	+ 33.6	− 28.5	+19.3
K											
Large	255.4	+17.9	+46.	+178.	− 59.	256.9	+ 3.9	+ 31.	+131.	− 9.	+14.0
Medium	269.8	+46.4	+ 0.2	+ 58.9	− 61.8	245.5	+35.9	+ 14.6	+ 31.9	− 25.4	+10.5
Small	267.6	+72.4	+ 0.5	+ 12.2	− 38.8	282.2	+66.2	− 3.2	+ 14.7	− 34.1	+ 6.2
Mean	.....	+42.6	......	....	....	......	+32.7	....	....	....	+ 9.6
						Omitting ()	+29.3	....	....	....	+13.8
Adams' 500 Radial Velocities:											
Large	279.8	+33.6	−56.	+323.	−218.	275.8	+35.9	− 63.	+626.	−455.	− 2.3
Medium	281.9	+35.4	− 8.1	+ 38.2	− 27.7	202.8	+18.7	+115.2	+ 48.5	− 42.2	+16.7
Small	279.4	+70.7	− 1.2	+ 7.2	− 20.9	230.9	+50.9	+ 6.0	+ 7.4	− 11.7	+19.8
Parallax observed:											
0"06 and over	253.7	+47.9	+97.	+332.	−383.	258.0	+24.9	+186.	+871.	−414.	+23.0

The asymmetry found in the proper motions of the B stars, together with other peculiarities observed, prompted the making of separate solutions from the northern and southern stars, the results of which are given in Table III. These results show clearly the decrease of D with increase of μa and in addition, in all but

[1] *Astrophysical Journal*, **44**, 114, 1916.

[2] *Ibid.*, **26**, 187, 1911.

two cases,[1] *an appreciably larger D from the northern than from the southern stars.* Taking the differences between the general means of the north and south D, including those of Adams' list, we find

$$\text{Northern} - \text{Southern} = +11^\circ\!.3.$$

Omitting Adams' list because of the lack of southern stars, and rejecting the large discordant southern D, we have

$$\text{Northern} - \text{Southern} = +13^\circ\!.3.$$

These values are reasonably close to $+9^\circ\!.1$, that between the general solutions from radial velocities and from proper motions; and to $+10^\circ\!.6$, between similar simple means of D derived from radial velocities and proper motions according to spectral class, 1259 radial velocities, and 4686 proper motions.

The difference between the simple mean of D from the radial velocities and proper motions of the *same 1194 stars* (by spectral class) is $+13^\circ\!.8$. This indicates that the differences in D between northern and southern stars from proper motions is fundamental and exists in the brighter stars (1194) as well as in the fainter stars in Boss's solution, and that the discordance in D between the radial velocities and proper motions is not due to the 3492 fainter stars.

It is now of interest to examine the positions of the apex derived from the northern and southern radial velocities separately, as given in Table IV.

With the exception of the K and M stars of magnitude 3.0 and fainter, the individual values of D from both northern and southern regions agree quite well, and their means to the nearest degree. The difference for these K and M stars is so large and so consistent that it is difficult to believe that it is not to some extent real, when we consider the considerable number of stars in the K type.

In comparing the mean D from the radial velocities of the 1194 stars with those from the proper motions of the same stars, *the significant thing appears to be that the southern stars give closely*

[1] An examination of the data in the A, F, G medium (southern) group showed several abnormal stars. Rejecting four of these brought the D slightly below that from the northern stars.

the same D from both, whereas for the northern stars the apex from the proper motions averages 20° to the north of that from the radial velocities.

TABLE IV

APEX AS DERIVED FROM RADIAL VELOCITIES OF NORTHERN AND OF SOUTHERN STARS

CLASS	NORTHERN STARS			SOUTHERN STARS		
	A	*D*	Number of Stars	*A*	*D*	Number of Stars
All classes: 2.9 and brighter	251°	+44°	50	258°	+40°	60
B	254	+27	70	288	+31	123
A	258	+20	97	264	+11	88
F	271	+ 9	79	264	+13	105
G	267	+19	64	249	+23	67
K	273	+ 8	151	268	+36	240
M	269	+15	24	272	+41	41
Mean	263	+20	....	266	+28	....
Mean omitting M	262	+21	....	265	+26	....
Mean omitting K and M	260	+24	....	265	+24	....

These values of *D* from the 1194 stars of the spectral classes B, A, F, G, and K are given below. Class M was omitted for the present because of its relatively few stars.

	DECLINATION OF APEX FROM	
	Northern Stars	Southern Stars
Proper motions	+43°	+29°
Radial velocities	+21	+26
P.M.−R.V.	+22	+ 3

What the declinations of the apex are for the *two regions separately* from the proper motions of the 4686 stars is not known. It may be noted, however, that the average *D* of the six spectral classes of Table II is +35°.6, as against +34°.3 obtained by Lewis Boss from a general solution from all of the stars, and that the average *D* from the proper motions of the 1194 stars of Table III (omitting one) is +36°.0. There is no evidence here that the same peculiarity in the proper motions in the two hemispheres does *not* extend to the 3492 stars as well.

If we examine the rectangular co-ordinates resulting from the solutions (Table III) we find that *nine out of the entire twelve values*

of Y and all of the values from the small $\mu\alpha$ are smaller for the northern than for the southern stars. In other words, this indicates a smaller average *parallactic* displacement for the northern than for the southern stars. This difference decreases from stars of class B, where it is large, to the K stars, where it is slight.

If now, to illustrate, we take the results from the B stars, and *use the value of Y from the southern stars* to determine the *D* for the *northern* stars, we shall obtain essentially the same *D for both northern and southern stars*, a value which agrees closely with the *D* obtained from the radial velocities of the same stars and with that from radial velocities of all of the spectral classes. This will be true in principle also for the other cases.

As to the interpretation to be put upon this asymmetry in the proper motions, attention may be called to two other facts which bear strongly, viz.: (*a*) the solar velocity deduced from the radial velocities of the northern stars is consistently less than from the southern stars; (*b*) the proportion of stars with contrary parallactic $\mu\alpha$ is greater for northern than for southern stars.

The most reasonable interpretation of the three distinct sets of appearances above would seem to be that the parallactic asymmetry is real and that the stars in question of the northern sky are moving in general with the sun.

These conclusions are of course tentative, for any matter as obscure as the general motions of the stars, closely associated as they are with the motion of the observer, neither of which is accurately known either in direction or in amount, must of necessity require a most thorough investigation from all possible points of view and with more data, particularly for the fainter stars, than are now available. I am aware that the peculiarities discussed in this paper are susceptible of other interpretations. Taken altogether, however, it seems to me that a very simple and probable explanation of all of these observed phenomena is suggested by the forms and appearances of a class of celestial bodies which is by no means rare. I have no hesitancy, therefore, in expressing the belief that the evidence which seems very direct is sufficient to justify the assumption as a *working hypothesis* that the general motions of our stellar system are either circular or spiral.

Other peculiarities have been observed, particularly in the distribution of the residual radial velocities, which appear to be most significant and will, as soon as a fairly trustworthy general idea of the system is obtained, aid materially in improving our knowledge of the details. Fuller discussion of many points connected with the questions raised in this paper is desirable, but it seems to me that it should await other related investigations and a general advance of the whole subject.

The foregoing explanation of the cause of the difference between the positions of the solar apex from radial velocities and from proper motions indicates that the radial velocities yield the most representative position on the whole, and that until the peculiarities observed in the proper motions, particularly in the northern sky, have been thoroughly investigated, positions depending upon them alone must be adopted with especial caution.

Great care is necessary as yet in considering any facts relating to the solar motion, for all of the results so far obtained appear to be influenced by the underlying motions of the stars themselves and more than ever to emphasize its purely relative character.

CONCLUSIONS

I. The position of the apex of solar motion depends upon the proper motions in right ascension of the stars used. The differences appear to be greatest in declination, the stars with large proper motion yielding apices south of those from the stars with small proper motion. This effect is shown in the results both from proper motions and from radial velocities.

II. The differences in D of the apices of solar motion as derived by other investigators from radial velocities and from proper motions are consistent and they appear in general to be greater for the stars of late type than for those of early type. This discordance appears to arise chiefly from the proper motions of the northern stars and to be satisfactorily explained by the assumption that the parallactic displacement of the stars is systematically less in the northern sky.

Observatorio Nacional Argentino
Córdoba
October 1, 1916

THE CAUSE OF THE SO-CALLED POLE-EFFECT IN THE ELECTRIC ARC

By T. ROYDS

Differences of vapor-density were first suggested in *Kodaikanal Observatory Bulletin*, No. 38, as the cause of the displacements of certain lines in different parts and conditions of the electric arc, and of the abnormal sun-minus-arc displacements of the same lines. Since, however, direct experimental proof is wanting, and has been said to give negative results, it seems desirable to discuss the evidence and experiments at the point at which work here on the subject has to be abandoned.

The cause of the displacements in the electric arc has also been treated by St. John and Babcock,[1] Gale and Whitney,[2] and Whitney,[3] none of whom discusses the evidence and conclusion in *Kodaikanal Observatory Bulletin*, Nos. 38 and 40.[4]

In the last two papers on experiments with a calcium arc, the pole displacement is ascribed to the greater amplitude of vibration of the electrons, and is said to depend on the intensity-gradient along the arc. The latter phrase is unfortunate, as, so far as I understand them, the authors do not mean the rate of change of intensity, but intensity-differences.

It must be obvious to every experimenter that the intensity of lines is great in those regions of the arc where displacement occurs; but, as it is equally true of lines which do not undergo displacement and of those which are displaced to the red and to the violet, one fails to see how the displacement can be said to depend on the intensity-differences. One might with equal or more truth say that the displacement depends on the width of the spectral lines, or on their diffuseness, but for reasons which have already been

[1] *Astrophysical Journal*, **42**, 231, 1915.

[2] *Ibid.*, **43**, 161, 1916.

[3] *Ibid.*, **44**, 65, 1916.

[4] Both these bulletins appeared in 1914.

elaborated,[1] I believe that the displacement depends on the unsymmetrical character of the spectral lines. I have not met with a single case where lines whose character was known were not displaced, either not at all, to the red, or to the violet, according as they were symmetrical, unsymmetrically widened toward the red, or unsymmetrically widened toward the violet, *except under those conditions, e.g., in reversals, where the vapor-density has been kept low.* Of course these phenomena—unsymmetrical character, intensity, etc.—are not the *cause* of the displacement, but are attendant effects due probably to the same cause.

Increased amplitude of vibration of the electrons is suggested by Gale and Whitney[2] as the cause of the displacement in the electric arc, but it is easy to see that this cannot be. The most effective and probably the only certain way known to me of increasing the amplitude of vibration of the electrons in the atom is to raise the temperature, but the displacements in the arc are not an effect of temperature, for many reasons, among which the three following seem sufficient.

1. Little is known of the variation of temperature along the arc, but it is certain that the positive pole is much hotter than the negative, whereas under normal conditions the displacement is greatest near the latter. The enhanced lines, which are high-temperature lines, appear stronger at the positive pole than at the negative,[3] also indicating that the temperature is higher there than at the negative pole.

2. The experiments described in *Kodaikanal Observatory Bulletin*, No. 40, and here, show that the displacement at the negative pole can be varied to any desired extent without reason for believing that the temperature of the arc is altered in any appreciable degree.

3. In the sun's reversing layer, where the temperature exceeds that attainable in the arc, the displacement of lines unsymmetrical in the arc is in the direction *opposite* to that of the displacement at the poles of the arc.

[1] Royds, *Kodaikanal Observatory Bulletin*, Nos. 38, 40.

[2] *Loc. cit.*

[3] A. Fowler, *Monthly Notices*, 67, 154, 1907.

Although the evidence given in *Kodaikanal Observatory Bulletin*, Nos. 38 and 40, is strongly in favor of density as the cause of the displacements, there are many difficulties in the way of direct experimental proof, due primarily to the difficulty of controlling the vapor-density in a source of light. Experiments with different quantities of material, such as those giving Gale and Whitney's Tables III and IV, fail; or at any rate are inconclusive, because there is no reason to believe that the atoms have been separated a greater distance with the smaller amount of material. If the atoms are vaporized in clusters, they may not be removed from each other's influence any more than when a larger amount has been used. Exposure-times are not a sufficient test of vapor-density, but only an indication of the total amount of material consumed.

On account of this difficulty it was thought better to use alloys as electrodes. Presumably the molecules in an alloy are so intimately mixed with another metal that each would be surrounded by molecules of another kind, and would be removed from the influence of those of the same kind. Even so, the experiments gave negative results. The best alloys available were the coins of the Indian coinage, the silver coins containing 10 per cent of copper, and the nickel coins containing 20 per cent. As the silver coins, and the money they represent, melt away rapidly and do not give a steady arc, the experiments were conducted mostly with nickel coins (value one anna). With a nickel coin as one electrode, and the other another coin, iron, or carbon, the wave-lengths and displacements of the three copper lines $\lambda\lambda$ 4480, 4509, and 4531 were compared with those in the arc between copper electrodes with the same length of arc and current-strength. The maximum displacement of the first and last was about $+0.05$ A and of the second about $+0.025$ A. The wave-length at the center of the alloy arc was identical with that at the center of the pure copper arc, but it was found that the wave-length at the negative pole could be varied at will by varying the material of the negative electrode. With carbon as negative electrode and the nickel coin or copper as the positive, the displacement of the copper lines at the negative electrode could be made very small, especially with those conditions when the green luminosity surrounding the positive

electrode did not reach up to the negative which showed the characteristic blue of the carbon arc. When the nickel coin is negative, and the positive pole a coin, iron, or carbon, there is on the other hand not the slightest difficulty in obtaining displacements at the negative quite as large as those at the negative pole of the arc between two copper electrodes.

The results with the alloy were therefore, in the main, disappointing, especially the fact that the wave-length at the center of the alloy arc was identical with that at the center of the pure metal arc. There is, however, one case, the sodium pair $\lambda\lambda$ 5682, 5688, where it is possible to obtain a displacement in the same direction as that in the sun and opposite to that usual at the negative pole. The data are given in *Kodaikanal Obersvatory Bulletin*, No. 40. The solar displacement of these lines, comparing the center of the sun's disk with the center of a very long arc, is -0.14 A (i.e., to the violet), the displacement of the unreversed line at the negative pole is $+0.36$ A, while the displacement of the *reversal* which occurs at the negative pole is -0.019 A. The sodium pair is very sensitive to displacement, and is only a case more extreme than many others, found in the bulletin referred to, for which the displacement at the negative pole is much smaller if the line undergoes reversal there than if the line remains unreversed. A new example of this has turned up in the first subordinate triplet of calcium near λ 4450. If the lines are reversed at the negative pole, the displacement is quite small or zero[1] and the lines appear almost, if not quite, symmetrical.[2] If, however, the lines are obtained unreversed at the negative pole, the displacement amounts to about -0.012 A and the unsymmetrical widening toward the violet is evident.

I have not met with cases such as that recorded by Whitney, where the displacement of the reversal was identical with that of the unreversed lines (it is not so for these lines on my photographs), but there is nothing impossible in it on the density hypothesis.

[1] Royds, *Kodaikanal Observatory Bulletin*, No. 40.

[2] Royds, *Astrophysical Journal*, **41**, 154, 1915, and *Kodaikanal Obseratory Bulletin*, No. 43.

The only way in which the results with the alloy and pure metal can be reconciled with the density hypothesis is to suppose that the density-differences effective in producing the displacements are of a much higher order than those obtained using the alloy containing 20 per cent of the metal investigated. It would seem that the atoms influence each other only soon after they are torn off from the electrode, as if they occur there in compact clusters which are soon dissipated so that when they reach the center of the arc the atoms are removed out of each other's influence. In the sun the density is supposed to be so small that a farther separation and displacement takes place. If the density-effect is due to the mutual electrical fields of the atoms, it is conceivable that fields of atoms of a different kind would also have an influence, thus explaining the considerable displacement at the poles of the alloy arc compared with the pure metal arc.

The considerations of the last paragraph would also explain the negative results of St. John and Babcock, but it cannot be conceded without further information that increasing the quantity vaporized increases the vapor-density in the furnace in the same ratio. One would have thought that the greater the quantity of material vaporized the greater would be the rate of its removal by condensation on the cooler parts of the tube.

The really interesting result of Gale and Whitney's and of Whitney's experiments is that they have, apparently, succeeded in obtaining arc conditions which bring the normal displacement at the negative pole of the arc down to zero, and even, for the more sensitive lines in the direction opposite to the usual one, i.e., in the same direction as the sun-minus-arc displacement.

I agree with Duffield's remarks on the influence of density- and temperature-gradients in light-scources on the displacement of spectral lines,[1] but I should like to make clear that the gradients cannot have any influence unless density and temperature are themselves causes of displacement.

Although direct experimental proof has not been obtained, I cannot find any hypothesis other than density to explain the displacement of certain spectral lines in different parts and conditions

[1] W. G. Duffield, *Philosophical Magazine*, 30, 385, 1915.

of the arc and the abnormal sun-minus-arc displacement of the same lines which have been discussed in ***Kodaikanal Observatory Bulletin***, Nos. 38 and 40. Whether some additional conditions are necessary, or whether the density-differences effective in displacing lines are larger than those hitherto attempted, are points for further experiment. It is hoped that a source of light may be constructed where the vapor-density can be varied over a large range when it is possible to resume these experiments.

Kodaikanal Observatory
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STUDIES BASED ON THE COLORS AND MAGNITUDES IN STELLAR CLUSTERS

By HARLOW SHAPLEY

FIRST PART: THE GENERAL PROBLEM OF CLUSTERS[1]

An extensive investigation of the magnitudes and colors of individual stars in globular and open clusters has been undertaken at the Mount Wilson Observatory. The discussion of the results will be published in the form of a series of papers, each article dealing with a definite problem, or with the observations on a single cluster. The first communication, of which the present note is an abstract, is intended to serve as a brief introduction, reviewing our knowledge of cluster systems and treating of the value and general bearing of the investigation of clusters on the problems of stellar astronomy.

1. *The classification and distribution of clusters.*—A stellar cluster is generally defined as any stellar group, the members of which are known to be physically connected, or which from their apparent positions may be assumed to form a distinct physical system. This broad category includes the Hercules cluster as well as the Ursa Major group (or even a system of groups such as our whole galactic domain). Various classifications have been proposed, but, because of the great variety of types and the gradual transition from one type to the next, none is much more than a working convenience. Clusters have been classed as loose, irregular, ragged, coarse, open, furrowed, globular, condensed, nebulous, etc., with many qualifying adjectives for each group, but practically all designations are based on superficial appearances.

It is proposed to adopt in the present series of papers the following working classification: (1) globular clusters; (2) open clusters; (3) moving clusters.

[1] Abstract of *Contributions from the Mount Wilson Solar Observatory*, No. 115, dated August 1915.

Although they are not mutually exclusive, there are some very good reasons which will be brought out in the later discussion for holding rather strictly to these three classes. The first class has for its type the Hercules cluster (M 13) or Messier 3, and for its distinguishing marks great condensation toward the center and numberless stars. The second contains clusters similar to Messier 11 or Messier 67 or even Praesepe, thus exhibiting a wide range in condensation and in number of stars. The third division includes the Taurus group, the Pleiades—in fact, any cluster of stars showing communal motion.

The adopted classification has on its surface the same lack of definite limits as other proposals; but in distribution we can discriminate sharply between open and globular clusters, and in probable distance we can temporarily, at least, differentiate moving groups from other open clusters. The apparent distribution in space is so striking that, taken in connection with other characteristics just coming to light, we are led to believe that there is a fundamental difference between the two main classes—the condensed globular objects and the open clusters.

Among others, Bailey, Bohlin, Hinks, Perrine, and more recently Melotte have discussed the distribution of clusters with respect to the galaxy. The most conspicuous result is that the open clusters are almost exclusively in the Milky Way and symmetrically arranged with respect to it, while the globular clusters are widely distributed in galactic latitude and quite limited in galactic longitude. The globular clusters are mostly in one hemisphere, the pole of which, according to Melotte, is at galactic latitude -9°, galactic longitude 296°. It should be noted, moreover, that no other class of celestial objects shows so marked a dissymmetrical arrangement with respect to the galactic plane.

2. *Previous observational and theoretical studies.*—Practically all the detailed observational work on individual clusters, with the exception of the investigations of cluster variables, relates to counts of the stars and to determinations of positions. No serious attempts have been made to determine accurate magnitudes, chiefly because of the lack of dependable magnitude scales for the fainter stars.

The more important investigations of clusters can be very briefly summarized. The most valuable contributions have been made by Bailey and by von Zeipel. The former has accumulated vast amounts of important information relative to cluster variables (practically all we know of these objects, in fact), as well as valuable data on classification, distribution of stars in clusters, and of clusters in the sky. In the study of Messier 3, von Zeipel has published the largest of all cluster catalogues, and has made investigations of considerable importance on the distribution of the brighter stars in several clusters, and on the possible physical relationships of the individual stars. E. C. Pickering, H. C. Plummer, See, Eddington, and Strömgren and Drachmann have contributed to the theoretical interpretation of the arrangement of stars in clusters.

The spectra of the brighter stars in a number of the brighter, very open clusters were classified by Mrs. Fleming. Of the spectra of individual stars in the fainter open clusters and in globular clusters, nothing is known aside from the work of Pease and Adams, who have classified provisionally 40 spectra in Messier 13. All spectral classes from A to K were recorded, with an average of type F, agreeing with the work of Fath and of Slipher on integrated spectra of various globular clusters.

It has been frequently suggested that certain clusters are relatively yellow, others white. But the first definite observation of color was made qualitatively by Barnard, who compared photographic magnitudes with visual and photo-visual estimates for a number of stars in the Hercules cluster.

3. *Correlation between diameters and brightness in clusters.*—From Bailey's *Catalogue of Bright Clusters and Nebulae*, which may be considered homogeneous over the field it covers, we obtain the data in Table I relative to mean diameters of globular clusters (outside the Magellanic Clouds) and the magnitudes of the brightest stars in each.

The striking result that magnitude increases with decreasing apparent diameter is to be expected if distance is correlated with apparent size. It suggests at once the possibility of deriving

relative mean parallaxes for clusters from measures either of diameters or of magnitudes.

4. *The purpose of the present study of clusters.*—The object in taking up the investigation of the magnitudes and colors in stellar clusters is twofold. First, it is hoped that considerable advance can be made in our understanding of the internal arrangement and physical characteristics of these objects. Secondly, and probably of more importance, it is hoped that through the acquisition of a clearer appreciation of stellar clusters, particularly of globular systems, some contribution can be made to the knowledge of our own galactic system. For it is quite obvious that a globular cluster, whether considered as entirely extraneous to our galactic domain or not, is in itself a stellar system on a great scale—a stellar unit which without doubt must be comparable to our own galactic system in many ways, though possibly differing from it fundamentally in others.

TABLE I

Brightest Magnitude	Number of Clusters	Mean Diameter
9	1	20′
10	0	....
11	1	18
12	7	9.4
13	25	10.0
14	5	4.4
15	3	3.3
16	1	3

A. In the study of clusters, the results which may be derived only from a knowledge of magnitudes and colors are in part as follows: (1) laws of distribution or density for different magnitudes and colors; (2) relation connecting absolute magnitude and color, and its variation from cluster to cluster; (3) total number of stars in the faint globular clusters predicted from observed luminosity-curves; (4) dependence of stellar variability on color and magnitude; (5) change of color with light-variation, and the consequent opportunity of investigating on a large scale the causes underlying Cepheid variation.

B. In the study of the galactic system the advantages of an investigation of cluster magnitudes and colors include the following: (1) elimination of the factor of distance-selection; (2) the possibility of dealing with complete stellar systems—unfortunately not literally true for globular clusters, because of the great number and extreme faintness of the stars; (3) the availability of large numbers for statistical discussion; (4) the possible evaluation of absorption of light in space and of color-factors depending on absolute magnitude.

C. In addition to those suggested above the following general astronomical problems are closely connected with the present study of clusters: (1) frequency of absolute magnitudes and spectra in stellar systems—luminosity-curves; (2) the order of stellar evolution; that is, the probable character of the progression of spectral type (color) with age; (3) the distances of clusters; (4) relation of the globular cluster to the open cluster and to the galaxy.

5. *Remarks on the methods of magnitude determinations.*—The derivation of photographic and photo-visual magnitudes, and hence also of the colors of stars in clusters, has been made possible through two important factors. The first is the 60-inch reflector of the Mount Wilson Observatory; the second and more necessary one is the fundamental work by Seares in establishing photographic and photo-visual magnitude scales for the faint stars.

The 60-inch reflector allows the speed necessary for successful intercomparison of the fields of faint stars, and at the same time the scale, even at the principal focus, permits the study of the stars at the very center of many of the globular clusters.

The magnitudes determined with the reflector from focal images make no claim to high accuracy. And, indeed, for the present at least, high accuracy is not at all requisite. It is sufficient if we obtain color-indices with a probable error of a tenth of a magnitude. The crucial point is not the size of the accidental errors nor the precision of the brightness for any given cluster star, but rather the unimportance of the systematic errors and the availability of dependable magnitude scales over the entire interval of brightness investigated.

SECOND PART: THIRTEEN HUNDRED STARS IN THE HERCULES CLUSTER (MESSIER 13)[1]

1. *Preliminary remarks.*—The first paper of this series contains a brief general summary of previous investigations of stellar clusters and an outline of the object, methods, advantages, and limitations of the proposed determination of magnitudes and colors at the Mount Wilson Observatory. The second communication, which is abstracted in the following pages, treats of the results for the Hercules cluster. This well-known system was considered first for several reasons: (1) its position in the sky when observations were begun was favorable for comparison with the Pole; (2) two good catalogues of the positions of the brighter stars were available; (3) only two variable stars had been recorded among the thousand brightest objects—an obvious advantage in deriving colors from non-simultaneous determinations of photographic and photo-visual magnitudes; (4) spectra of about forty of the individual stars were known approximately from direct classification.

No other globular cluster has been studied so frequently and extensively as Messier 13. It was one of the earliest placed on record,[2] being discovered accidentally by Edmund Halley 201 years ago. Messier saw the Hercules cluster as a brilliant circular nebula, but the elder Herschel resolved it partially in 1783, and later completely with the 20-foot telescope. An account of the earlier studies by Messier, D'Arrest, and others is given by Scheiner in his monograph *Der Grosse Sternhaufen im Herkules Messier 13*.

Formerly, in the drawings and verbal descriptions, and later in the early photographs, much attention was paid to the dissymmetrical arrangement of the brighter stars in the cluster and to the possible existence of visible nebulosity. Contributions to these

[1] Abstract of *Contributions from the Mount Wilson Solar Observatory*, No. 116, dated August 1915.

[2] Messier 22 was discovered by Ihle in 1665; ω Centauri was noted as a lucid spot in the sky by Halley in 1677, though it was previously known to Bayer as a hazy star, and even to Ptolemy as the star in the Cloud on the Horse's Back; in 1681 Kirch discovered Messier 11 (not globular, but rich and considerably condensed) and Messier 5 in 1702. All of these earlier discoveries consisted in noting that the objects were nebulous spots rather than single stars. The resolution into clusters awaited in most cases the telescopes of the Herschels.

subjects have been made by Secchi, the Earl of Rosse, Trouvelot at Harvard working with Winlock, and Harrington at Ann Arbor assisted by a professional artist. Holden reported 13 different intersections of distinct lanes, Schaeberle interpreted the arrangement as forming a spiral with one arm clockwise, the other counterclockwise, while Scheiner and Barnard have voiced conservative views relative to the structure and nebulosity, respectively. Palmer, Miss Clerke, Perrine, and Ludendorff have also discussed different phases of these matters. The conclusion from all the former studies, which is strengthened by the results from plates made at Mount Wilson, is that no nebulous material can be detected in the Hercules cluster (at least unless it is fainter than the twentieth magnitude), and that the so-called "channels" among the brighter stars, while apparent in some form to the eye and on photographic plates of the proper exposure and contrast, do not appear among the fainter magnitudes and are probably to be considered of no particular importance in the structure of the cluster.

The first attempt to record the positions of the individual stars with sufficient accuracy to give future observers a trustworthy basis for the study of proper motions was made by Scheiner in 1891–1892. His catalogue contains 833 stars—all that were measurable on two plates with exposures of 2 hours and 1 hour, made with the 32.5 cm refractor of the Potsdam Observatory. A second catalogue of somewhat higher accuracy was made by Ludendorff, and contains 1136 stars. It is based on two plates taken with the same instrument at Potsdam in 1900 and 1902, but with exposures of 2.5 and 2 hours, respectively. Comparison of the positions derived by Scheiner and Ludendorff show no definite proper motion during the interval of ten years. In none of these investigations has serious consideration been paid to the determination of magnitudes.

E. C. Pickering determined the distribution of stars in Messier 13, and some valuable theoretical considerations of his results are contributed by H. C. Plummer. A much more exhaustive consideration of the star-density is made by von Zeipel, who used copies of plates made at the Yerkes Observatory and also, for the outer portions of the cluster, photographs made with the Upsala

refractor. About 2500 stars were discussed, but only roughly from the standpoint of brightness. Of the many important results and conclusions derived by von Zeipel for this and other globular clusters only the following need be mentioned at this point:

1. From the ratio of the number of bright stars to faint stars in different regions it is necessary to conclude that the physical state near the center differs from that in the exterior regions.

2. So far as these brighter stars are concerned, the density in Messier 13 shows that the cluster is comparable to a spherical nebula (gaseous sphere) in convective equilibrium conforming to certain adiabatic laws.

3. It is entirely possible that the globular clusters have attained their adiabatic equilibrium solely by virtue of the mutual attraction of the constituent stars.

4. Each cluster is a gigantic system containing approximately a million stars.

This last conclusion is admittedly quite speculative. It is based upon purely theoretical assumptions that now appear to be inadmissible; but it will be shown from other considerations, which have a good observational basis, that the conclusion itself may not be far wrong. Later it will be shown why No. 1 above is a sound result, other evidence being presented to show that conditions may exist in the center of the cluster which are not evident in the outer regions. This same evidence will suggest that near the center, at least, von Zeipel and others are dealing with a distribution which is not real, and that the theoretical results, including Nos. 2 and 3 above, must be modified accordingly.

The integrated spectrum of Messier 13 has been observed by Fath at the Lick Observatory and at Mount Wilson. The work by Pease on the spectra of individual stars in the cluster is discussed in a later section of this paper.

2. *The observational data.*—The magnitudes are derived from the complete measurement of two pairs of plates, made on different nights and each consisting of one isochromatic and one ordinary photographic plate. In order to avoid, in the derivation of color-indices, errors depending on the presence of undetected light-variations in some of the stars, the plates of a pair are always taken

in succession so that only uncommonly rapid light-changes can affect the results. It is believed that errors arising from this source are negligibly infrequent and small. The results are indirectly based on the partial measurement of two other pairs of plates; and two plates containing each a long and a short exposure have been used to determine the magnitudes of fainter stars. Some preliminary plates have been of service in deriving magnitudes of newly discovered variables. The observations were made with full aperture at the principal focus of the 60-inch reflector.

TABLE II

AVERAGE PROBABLE ERROR OF A MAGNITUDE DETERMINATION

	Photographic		Photo-visual	
	Whole Cluster	Distance>2′	Whole Cluster	Distance>2′
Probable error:				
One plate	±0.068	±0.057	±0.046	±0.044
Two plates	±0.048	±0.041	±0.033	±0.031
Corrected for systematic deviations:				
One plate	±0.061	±0.044	±0.040	±0.035
Two plates	±0.043	±0.031	±0.029	±0.025

The plates have been measured chiefly by the method described by Seares in *Mt. Wilson Contr.*, No. 80, Section 6. Supplementing the direct measurement with the graded scale, the writer has found it of great advantage to carry on simultaneously a series of estimates—an independent visual determination of the relative brightness of adjoining images by the Argelander method, using as steps tenths of intervals between scale-images.

In addition to this double measurement of the plates, series of completely independent duplicate measures were made to test for the various suspected systematic errors. Effects of fatigue and other personal-equation errors were detected early and avoided. During measurement the plates are illuminated artificially, so that variations in weather and sky illumination disturb the work little, if at all.

3. *Discussion of errors.*—From 1051 photographic and 244 photo-visual residuals were derived the probable errors summarized in Table II. The average probable error is less than ±0.05

mag. for 95 per cent of the magnitudes used in the statistical discussions. This error includes not only uncertainties of observation in the cluster magnitudes, but also the errors in the polar standards, uncertainties in the distance correction and zero-point, and other plate errors. When the individual residuals are corrected for systematic deviations, thus excluding the plate errors, the probable errors become as in the last two lines of the table.

The average probable error of a tabulated color-index (without correcting for systematic deviations) varies from ± 0.082 to ± 0.058, and for about 75 per cent of the stars is ± 0.067. Excluding stars within $2'$ of the center the limits become ± 0.072 and ± 0.051, and the median value is ± 0.060.

The discussion of errors shows for the denser central region of the cluster, not only the expected larger accidental deviations, but also a systematic error apparently inherent in the plates themselves. At first it appeared not serious enough to introduce unallowable errors into the magnitudes and colors. Further investigation, however, and especially the statistical work on color-indices, suggests that a photographic phenomenon analogous to the one discovered and studied by Eberhard may influence the results to a considerable extent. Eberhard found that the intensity of a photographic image depends on the density of neighboring parts of the plate.

Further discussion of this phenomenon in its probable bearing on the magnitudes in the center of the Hercules cluster will be postponed to a later paper. For the present some reasons for suspecting its appearance may be enumerated: (1) the nature of the field photographed and the method of development; (2) the discrepancy between directly observed spectra and color-indices in the center of the cluster; (3) the conspicuous increase of average color-index in the central regions; (4) some laboratory experiments in copying and enlarging photographs of clusters.

The reasons cited for believing that a systematic photographic error may affect the magnitudes within $1'.5$ or $2'$ of the center, with obvious and slight changes in wording, are also valid reasons for believing that there is no such effect outside the crowded central regions. In addition we have still stronger assurance that

the magnitudes of stars more distant than 2′ are reliable in the fact that the Polar Standards, which are very generally distributed on all plates throughout the outer regions of the cluster, show no trace of systematic error depending on distance.

A quantitative correction for an Eberhard effect is not possible, for as yet very little is known about its amount, variation with plate emulsions, or its control. In view of this circumstance it

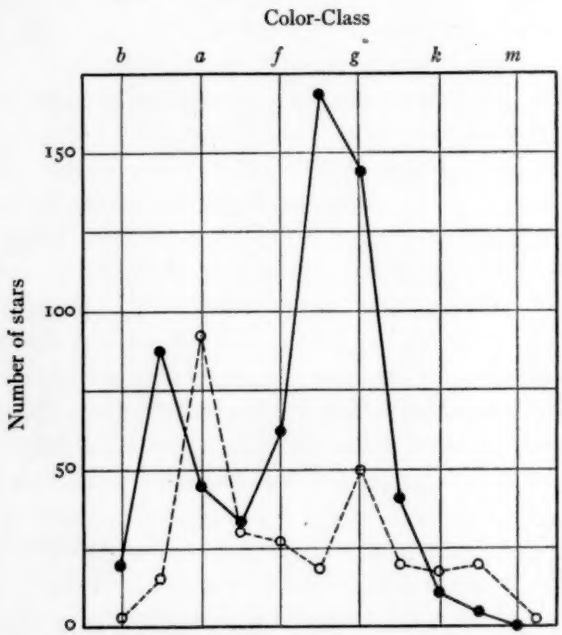

Fig. 1.—Frequency of color-classes in Messier 13 (full line), and in the sky at large

has been considered best to withhold publication of the magnitudes derived for the central stars of the cluster until later study can make them more definite. The data derived from them are retained to some extent, however, in the statistical tables and discussion, for it is through the analysis of this material that the peculiarity of the central region is best revealed.

4. *On the absorption of light in space.*—The catalogue of photographic and photo-visual magnitudes contains about 1300 stars. Owing to the probable presence of systematic error in the magni-

tudes in the denser central regions, the lists now published contain only about 650 stars; publication of the remainder awaits further investigation of suspected errors. Probably not more than 18 of the magnitudes refer to superposed non-cluster stars. Discarding incomplete or doubtful results, 1049 magnitudes and color-indices are available for statistical investigation, of which 495 are outside the central region. The range of observed photo-visual magnitudes is from 11.9 to 16.8; but the results are complete only down to magnitude 15.5.

The values of the color-index in the cluster range from -0.52 to $+1.96$, with the greatest frequency about $+0.90$. Excluding the zone within $2'$ of the center, the upper limit is $+1.45$, with greatest frequency near $+0.70$. A plot of the frequency of color-class for the outer portion of the cluster is shown in Fig. 1, together with the frequency-curve of spectral types at large, the data for which are taken from Parkhurst's *Yerkes Actinometry*. To plot Parkhurst's data the spectral types were interpreted directly as color-classes, and the numbers given by him were divided by two. Though differing in detail, these curves are similar in two important characters, notwithstanding considerable lack of homogeneity in the Yerkes results as regards distance and absolute brightness—each curve has a maximum for blue and for yellow stars, and the limits of color are practically the same for both.

Probably the most remarkable feature of the distribution of color-classes is not the relative paucity of *a*'s and early *f*'s, nor the great extent of the range in color, but rather the highly significant fact that there are any negative color-indices at all. Of the 495 stars at a greater distance from the center than $1\overset{\prime}{.}9$, more than 17 per cent have negative color-indices, and of these one-fifth are bluer than represented by color-class *b*5.

It is well known that a condition of this nature cannot exist if there is appreciable molecular scattering of light in space. It will be shown later that the parallax of the Hercules cluster cannot reasonably be greater than $+0\overset{\prime\prime}{.}0001$, and it may be but one-tenth of that amount. Using the most recent value derived by Kapteyn for the change in color-index due to space absorption, namely, $d=+0.003$ mag. per unit of stellar distance ($\pi=0\overset{\prime\prime}{.}1$), the absorption

should be of the order of three magnitudes at least, and the minimum color-index $+2.5$. Suppose we use the very recent and smaller value, $d=+0.0015$, determined by Van Rhijn from a consideration of the stars included in the *Yerkes Actinometry*. Even then all the color-indices in Messier 13 should exceed a magnitude.

It seems necessary to conclude that in the direction of the Hercules cluster the selective extinction of light in space due to molecular scattering is negligible. If we grant a color-excess of $+0.1$ mag. in the cluster, due entirely to space absorption, the value of d cannot exceed $+0.0001$ mag. per unit of distance, an amount entirely inappreciable in dealing with the distances of the ordinary isolated stars.

Whatever the effect of intrinsic luminosity on redness may be, we can safely assume for the present that it is not sufficient to counteract or conceal the absorption of light in space if the previously found values of this quantity are adopted. It is also apparent that, over the small range in apparent magnitude (and, therefore, in absolute magnitude) observed in the cluster, the luminosity factor can have no serious influence on the observed color-index.

The interpretation of color-class directly as spectral type receives a valuable verification through the work by Pease on the spectra of the individual cluster stars, but owing to the provisional nature of the data the check cannot as yet be considered complete. The discussion of the spectra and the relevant color-indices will appear in a separate paper. For the present we only observe that for the four stars with distance greater than 1.5 the excess of observed color over that to be expected from the spectral types is but $+0.08$ mag. and the error possible in the magnitudes and spectral classification of a single star may easily exceed a tenth of a magnitude.

5. *Miscellaneous results.*—Hardly less striking than the result which indicates the apparent absence of light-scattering in space is the unexpected relation brought to light by plotting magnitude against color for any or all of the regions in the cluster. From the main statistical table the data are collected for the summary in Table III, which gives for successive regions the average photo-

visual magnitude for each color-class. The number of stars involved in each case is in parentheses following the mean magnitude. (The last column of this table shows the central redness in a decisive manner.) The successive lines of Table III show that for all distances from the center the average magnitude is brighter for the redder color-classes than for the blue. Not only are the

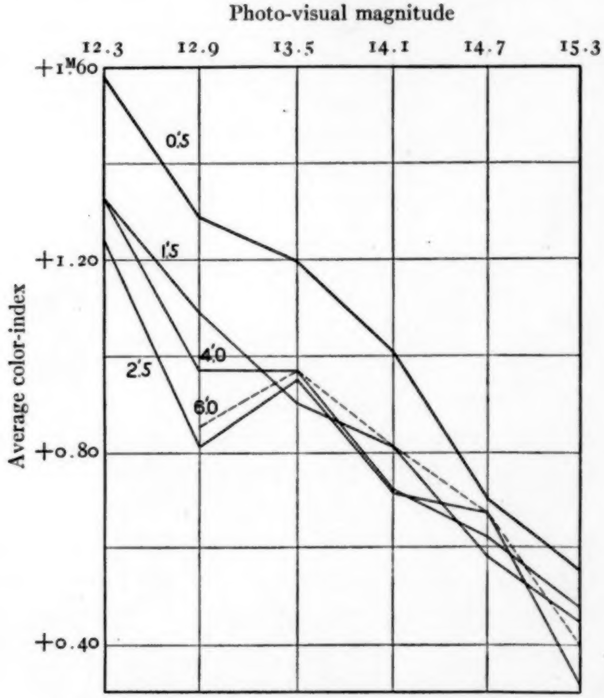

FIG. 2.—Decrease of color-index with decreasing brightness

g, *k*, and *m* colors associated in the mean with the brighter stars of the cluster, but the decrease of redness with decreasing brightness is distinctly progressive; in fact, nearly linear in all parts of the cluster. The phenomenon is illustrated graphically in Fig. 2.

For open clusters such as Messier 67 and the Pleiades, and for the stars in the sky at large, the relation between apparent or absolute magnitude and color has been found always in the sense of increasing redness with decreasing brightness. It is not necessary

to hasten the interpretation of the contrary result obtained for a globular cluster. For the present it will suffice to observe that we are now dealing with the three or four brightest magnitudes in a stellar system in which exists a range of absolute brightness of certainly more than 10 magnitudes. Doubtless we are discussing the giant stars and only the giants among a throng of hundreds of thousands. For the stars in the galactic system Russell has come to the tentative conclusion that the giants are of comparable absolute magnitude whatever their spectral type. The present result suggests that in this globular cluster, at least, the giants are brightest when reddest. The relation between absolute brightness and color is therefore the inverse of that for dwarfs, where, without much doubt, the cooling stars are growing redder and smaller with age.

Of the four hundred faintest stars measured in Messier 13, 85 per cent are bluer than color-class *g*0, which corresponds to solar spectral type; of the four hundred brightest stars 75 per cent are redder than *g*0. An analogous result holds for the outer regions alone. Between photo-visual magnitudes 16.0 and 16.8 only four out of a special list of a hundred faint stars have color-indices greater than half a magnitude, and none is found redder than the sun.

There is some evidence that the brighter stars are more strongly condensed in the center of the cluster than the faint ones; at least the average magnitude is brighter, even without allowing for the color-excess. Outside the central zone the cluster stars seem to be very thoroughly mixed with respect to magnitude and color.

The laws connecting star-density and distance cannot be determined until the phenomenon interpreted here as Eberhard effect (which also depends on distance from the center) has been thoroughly investigated; even neglecting this effect, the distribution laws ordinarily derived must be recognized as applying to only that very small percentage of all the stars which is represented by those of exceptionally high intrinsic brightness.

Five new variable stars have been found in the Hercules cluster during the course of the present study of magnitudes. Prior to this two were known, the first discovered by Bailey at Harvard, and

TABLE III

AVERAGE MAGNITUDES FOR DIFFERENT REGIONS AND COLOR-CLASSES

Distance from Center	Color-Class						All Colors	Average Color-Class
	b	*a*	*f*	*g*	*k*	*m*		
0'.00–0'.45	.....	.....	14.50 (9)	14.14 (16)	13.37 (24)	13.31 (12)	13.73 (61)	*k*1
0.46–0.95	14.48 (2)	14.83 (29)	14.55 (34)	14.06 (63)	13.64 (31)	12.57 (6)	14.17 (165)	*g*1
0.96–1.45	15.12 (10)	15.12 (33)	14.78 (48)	14.06 (76)	12.83 (7)	11.92 (1)	14.46 (175)	*f*7
1.46–1.95	15.26 (15)	14.98 (29)	14.74 (51)	14.12 (54)	13.19 (3)	12.08 (1)	14.57 (153)	*f*5
1.96–2.45	15.18 (29)	15.12 (14)	14.88 (38)	14.36 (36)	12.11 (3)	.....	14.76 (120)	*f*2
2.46–2.95	15.22 (17)	14.75 (12)	14.89 (32)	14.03 (31)	12.74 (3)	.....	14.58 (95)	*f*4
2.96–3.95	15.16 (12)	15.14 (17)	14.81 (51)	14.18 (33)	12.19 (2)	.....	14.67 (115)	*f*4
3.96–4.95	15.25 (16)	15.22 (11)	14.92 (31)	14.70 (20)	13.56 (4)	.....	14.91 (82)	*f*3
4.96–5.95	15.22 (6)	14.96 (5)	14.84 (18)	14.15 (20)	.....	.....	14.62 (49)	*f*5
$\geqq$ 5.96	15.16 (6)	14.03 (3)	14.74 (16)	14.54 (8)	13.69 (1)	.....	14.67 (34)	*f*5
All of cluster	15.19 (113)	14.99 (153)	14.87 (328)	14.15 (357)	13.34 (78)	12.96 (20)	14.50 (1049)	*f*6
Outside distance 1'.95	15.20 (86)	15.01 (62)	14.85 (186)	14.28 (148)	12.74 (13)	.....	14.71 (495)	*f*3

the second independently by Bailey and by Barnard. The periods of the two earlier variables are given by Barnard as 6.0 days and 5.10 days. Ordinarily they appear to be faint, evidently belonging to the Cepheid type. The range of variation is approximately one visual magnitude. For the five new variables the periods are not known; indeed, little is known of the light-variations beyond the fact that they appear definitely to exist. In every case, however, the smallest color-index is associated with the brightest photographic magnitude, which suggests that all seven stars are very probably Cepheid variables.

6. *On the probable parallax of the cluster.*—If, admitting a high degree of comparability between our galactic system and globular clusters, we assume that the brightest stars in Messier 13 have exactly the same absolute magnitude as the most luminous stars in our galactic system, we can compute the distance of the cluster as soon as we obtain the apparent magnitudes of the brightest stars in both systems and an assurance of negligible absorption in space. It would be better perhaps if we could assume equality of mean absolute magnitude for a number of stars of the same color-class in the two systems; or, if we could say, for instance, that the average B-type star has the same intrinsic luminosity wherever found. Then the determination of the distance of any cluster would await only measurements of magnitude.

In actual practice the derivation of the parallax of the Hercules cluster has not been simple or definite. The subject has been approached from many angles and the basic assumptions have been rather carefully examined. The complete discussion of the methods and results is deferred to a later paper, and for the present only a tabular summary will be presented.

The values in Table IV that are based on luminosity-curves fail of important weight because (1) of the provisional character of all theoretical luminosity-curves except for spectral Class B, (2) of the peculiar (perhaps chance) frequency of the brighter magnitudes among the early *b*-type stars in the cluster, and (3) of the possible necessity of distinguishing sharply between dwarf and giant stars in deriving luminosity-curves.

Incidental to the work on the Hercules cluster the following provisional parallaxes were obtained for similar objects:

Small Magellanic Cloud	$=0.''00006$
ω Centauri	$=0.00027$
Messier 3	$=0.0001$

7. *On the probable dimensions in the Hercules cluster.*—We conclude from the preceding considerations of the distance of the cluster that the parallax is certainly not in excess of $+0.''0001$. It may be but a tenth of this value, and some of the evidence would suggest still greater distance; but a parallax less than $+0.''00001$ is hard to reconcile with the magnitudes of the variable stars, if they are to be considered normal Cepheids. The provisionally adopted parallax, $+0.''00003$, is, however, more likely to be too large than too small. On the basis of this distance some computations relative to the probable dimensions of the cluster are summarized below.

TABLE IV

SUMMARY OF DETERMINATIONS OF PARALLAX FOR MESSIER 13

Method	π	No. Stars	Suggested Weight of $\frac{1}{\pi}$
I. From variable stars	$0.''00008$	2	4
II. From Kapteyn's luminosity-curves:			
C.I. -0.39 to -0.20	.000005	17	
" < -0.10	.000007	53	1
" " (Pv. mag. < 15.30)	.000009	23	
" -0.10 to -0.01	.00003	33	2
All colors	.00005	495	0
III. From Russell's data for absolute magnitude:			
C.I. < -0.10	.00005	53	4
All colors	$0.''00010$	495	1
Provisionally adopted mean	$0.''00003$		

The angular diameter of that part of the cluster measured for the catalogue is less that 16′. It is readily seen, however, from the long-exposure photographs, as well as from the table of densities, that the cluster extends far beyond this limit. Von Zeipel states, on the basis of plates made especially for the purpose of fixing the

limits, that the radius is not less than 17′. Accepting this result and the adopted value of the parallax, we find that the distance across the cluster is of the order of 1100 light-years. To an observer on the nearer edge of the cluster, a star on the opposite side would have a parallax of $+0\overset{''}{.}003$, and if intrinsically one hundred times brighter than our sun, would still be nearly two magnitudes below the limit of visibility to the naked eye. To an observer on the earth, however, the difference in the apparent magnitude of two such stars, one on the nearer, the other on the farther, side of the cluster, would be only 0.02 mag., and the difference in parallax negligible.

At the distance corresponding to the adopted parallax, 100,000 light-years, the sun would appear fainter than the twenty-second magnitude, and stars of the absolute brightness of Sirius would be fainter than the eighteenth magnitude. The absolute visual magnitude of the brightest star in the cluster would be -5.7. Several stars in the galactic system of this absolute brightness are already on record; and of the giants so far known there naturally must be a selection in favor of the nearer ones. We know, in general, only the fainter limit for the absolute brightness of many if not most of the galactic giants. For instance, a second-magnitude star with a parallax not greater than $0\overset{''}{.}001$ would have an absolute magnitude of -8 or brighter. Hence we could without seriously transgressing possibilities postulate absolute magnitudes of -10 or brighter. Since any such stars in the cluster do not exceed apparent magnitude 12.0, their distance must be nearly eight times greater than assumed above (perhaps of the order derived from the luminosity-curves of the early *b*'s) and the dimensions in the cluster would be octupled. But retaining the adopted value of the parallax, it is important to note that all the stars considered in the catalogues of Messier 13 have more than 200 times the solar brightness. In fact, probably no star as faint as the sun has yet been photographed in this cluster.

The long-exposure plates by Pease and by Ritchey would indicate, on the basis of the assumed distance, that there are probably more than 50,000 cluster stars intrinsically brighter than the sun. If all stages of stellar development are represented in the

cluster as we know them in our galactic system, we must believe that the total number of luminous stars is of the order of hundreds of thousands.

Assigning for illustration a diameter of 10,000 light-years to the galactic system, it would appear from the Hercules cluster to have an angular diameter of about 5°, perhaps comparing closely in general appearance with the Greater Magellanic Cloud as seen from the earth. Our system, to the observer in the cluster, would also resemble the Magellanic Clouds in the remarkable characteristic of being rich in variables. From the cluster the thousand or so brightest apparent magnitudes would be our stars of greatest actual light, and among them, as in globular clusters and in the Magellanic Clouds, there would be a remarkably high percentage of variable stars. All types would be present, moreover, for we know they are all of high intrinsic luminosity; but those most readily discovered, for obvious reasons, would be the short-period Cepheids; that is, the cluster-type variables.

8. *Suggestions toward some working hypotheses.*—Upon the basis of the foregoing computations and from the results of preceding sections of this paper we are led, for the guidance of future work on magnitudes in clusters, to suggest some working hypotheses relative to the form, extent, and distance of stellar systems. The proposed hypotheses will be modified, no doubt, as the work progresses; their purpose now is chiefly to represent collectively and briefly those ideas concerning general stellar problems that the present study suggests.

The peculiar distribution of globular clusters and their great distances from the earth show that they do not form a part of the galactic stellar system. Each globular cluster is a complete and separate system by itself. Apparently none of them is very near, compared to the outer limits of the galaxy. The stars of a few southern clusters, particularly of ω Centauri and 47 Tucanae, are brighter than those of Messier 13, and such systems may be much nearer than a hundred thousand light-years; but many clusters are considerably fainter and presumably more distant. In observing those similar to N.G.C. 4147 (if they are built on the same model as Messier 13) we may be seeing a million light-years into space.

Our galactic system is distinctly out of the center of gravity of this higher-order system of visible globular clusters.

Until contrary evidence is brought forward we may assume that all open clusters are parts of the galactic system. This is suggested by their galactic concentration, by their motions so far as now measured, by the number of stars they contain, and more directly by the evidence of their comparatively small distances shown in the magnitudes and colors of their component stars. These galactic sub-groups are often if not generally real physical systems. The Hyades at a greater distance would resemble one of the galactic clusters, but because of the relative paucity of members could never be comparable to Messier 13, as is sometimes suggested.

Some of the globular clusters may be comparable to the galactic system in size, form, or at least in stellar constituency. For the Hercules cluster closely comparable dimensions cannot be claimed. A more appropriate comparison would be between our stellar system and the larger Magellanic Cloud. That the maximum radius of the Milky Way is probably not greater than ten thousand light-years and may be somewhat less has been deduced from many lines of evidence, the most important of which is the color of the faint stars. The shortest radius of the system is variously estimated from one-half to one-sixth of the radius in the galactic plane.

Have globular clusters such planes of symmetry? From superficial appearances a negative answer is expected. The discussion of planes of symmetry in globular clusters involves at least three important factors—the degree, orientation, and nature of the supposed oblateness. (1) The ratio of the shortest diameter to the longest may be near unity, so that only refined study from any position in space (in or out of the cluster) would be able to detect the oblateness. (2) The cluster may be so oriented in space that, whatever the ratio of its axes of symmetry, the discovery of oblateness is impossible. (3) The absolutely brighter stars may not show the oblate form, the property of condensation toward a plane being confined to the fainter stars. This latter consideration is very relevant. In our system the naked-eye stars of the second spectral type show practically no galactic concentration. In globular clusters the brightest stars, which through their apparent

arrangement have given the type-name to the systems, are those that are absolutely bright and red.

Under the most favorable conditions of oblateness and orientation, therefore, the analogy to the galactic system should make the elongated form apparent only for the faint stars in the cluster. The concentration may and probably does depend on spectral type as well as magnitude. The colors in the Hercules cluster have been examined for the purpose of testing possible oblate distribution of the blue stars, but no decisive result was obtained. The counts in other clusters have also as yet yielded no positive contribution to this matter, dealing as they mainly do with only the brightest stars. A distinctly elongated or non-globular form, however, has been noted for two or three of the brightest clusters where fainter stars are involved in the photographs, and counts have verified the superficial appearance of a plane of symmetry in ω Centauri.

The conclusion is that globular clusters may very probably resemble our local stellar system in form, as well as in the properties of showing central condensation of stars, of containing all spectral types, including the giant red and yellow stars, and of having among their most luminous members a large quota of variables.

The base line used in the present case for the measurement of the amount of light-scattering in space is a very long one; those previously used have been relatively short. The hypothesis, based on the present result, that there is no appreciable space absorption, is therefore fairly safe.[1] Since it is not certain as yet, however,

[1] The coefficient of absorption can scarcely be one-tenth the most recent previous values, even with very generous allowance for error. That being the case, the result is intimately related to the discussion of the time-honored problem of the finiteness of the stellar universe. By universe in this sense we mean the system that is formed by stars of the kind that surround the sun and appear to us in all parts of the sky. Either the extent of the star-populated space is finite, or "the heavens would be a blazing glory of light," or there is an absorption of light in space, or there is a peculiar, non-uniform distribution of stars and star-systems throughout space (Charlier), or, except in our immediate vicinity, the stars as light-giving bodies are dead or not yet born. The last two clauses are usually not included in the discussion. Omit them. Then, since the heavens are not a blazing glory, and since space absorption is of little moment throughout the distances concerned in our galactic system, it follows that the defined stellar system is finite. There is nothing novel in this conclusion. It has been the prevailing one for a long time, and has been very easily derived by making the assumption, in spite of prevailing values of the coefficient, that space absorption is really negligible.

whether or not molecular light-scattering exists in the central part of the cluster, we must make the reservation that there is a possibility of some measurable space absorption for lower galactic latitudes. And of course we may expect patches of absorbing material here and there in space, as, for example, Barnard's dark nebulae in the Milky Way. In any case absorption should be treated as a varying quantity[1] depending on the condensation of light-scattering material, and not as a quantity that is constant, whether in the densest part of our galactic system or in the starless realm without.

The decision that molecular scattering of light is inappreciable in the direction of Messier 13 probably justifies the assumption that general non-selective space absorption (obstruction) is also of small importance. It also permits safer sounding of the depths of the Milky Way system in various directions, and, together with the absence of blue stars of fainter magnitudes, suggests that the limits have already been reached. There is a suggestion among the globular clusters that the stars at great distances from the center are intrinsically fainter than the nearer ones, because of either greater age, or smaller mass, or dependence of spectrum on distance. If such a condition exists in our galactic system we can better interpret the color-distribution of faint stars.

THIRD PART: A CATALOGUE OF 311 STARS IN MESSIER 67[2]

The open cluster Messier 67 was discovered about a century ago at Milan by Oriani. The early observations consisted mainly of general descriptions of the group by Messier and Sir John Herschel, and later of notes relative to the number and distribution of its members by Smyth and Fenet. More recently two photographic catalogues have been made of the positions of the individual stars, the first by Olsson at Stockholm in 1898, the second eight years later by Fagerholm at Upsala. In the positions of the 118 stars common to the two catalogues various systematic differences

[1] Seeliger has considered space-absorption both as constant and as decreasing with distance (*Sitzungsberichte der mathematisch-physikalischen Klasse der königlich-Bayerschen Akademie der Wissenschaften*, 1911, Heft II, p. 413).

[2] Abstract of *Contributions from the Mount Wilson Solar Observatory*, No. 117, dated May 1916.

appear, but there is no conclusive evidence of motion during the interval of eight years.

In the catalogues of positions the magnitudes are determined only approximately. The values derived by Olsson, evidently depending on rough estimates, are systematically too bright by more than two magnitudes. Fagerholm's magnitudes, on the other hand, are determined by careful measurements of diameters of stellar images; but they are based upon an extension of the visual scale of the *Potsdam Durchmusterung* and are systematically in error owing to the color of the stars.

In the present study the photographic magnitudes are given for all objects listed by Fagerholm, and for a few others which are within the zone covered by him but are missing from his table. Photo-visual magnitudes are determined for all but four of the stars within distance 12′ of the center. The average probable error of a color-index is about $\pm 0^{\mathrm{M}}.1$.

After discarding a few questionable values a total of 232 color-indices is available for statistical discussion. The average color-index is $+0.91$, with a strong condensation of color in classes *f* and *g*. No negative indices and but five less than $+0.4$ were obtained. All of the *m*'s and most of the *k*'s are near the center and the former are chiefly among the brighter stars, but aside from this no definite relation exists between color, distance, and magnitude.

From a study of these colors alone, the cluster appears to consist of but a hundred stars and to be of small extent, but an examination of the large star charts (Palisa-Wolf, Franklin-Adams) shows that the cluster's influence among the background stars may extend to a distance of nearly a degree.

There is much in common between the stars of the immediate background and those of the cluster—average color, magnitude, and condensation not appearing greatly unlike. In this respect, as well as in others, there is a wide difference between Messier 67 and the condensed globular system Messier 13, discussed in the preceding part.

MOUNT WILSON SOLAR OBSERVATORY
Abstract dated January 1917

Minor Contributions and Notes

ON POLE-EFFECT

In our paper on "Pole-Effect in a Calcium Arc"[1] Mr. Whitney and I had intended to mention the work of Royds and his theory that differences in vapor-density at different parts of an arc are the cause of the differences of wave-length, first discovered by Goos[2] and later studied by St. John and Babcock,[3] Royds,[4] and others.[5] But some of our results seemed to be contrary to Royds's hypothesis; in particular our results gave the same values for the wave-lengths of the Ca lines at the center of an arc between carbon poles which had been soaked in a dilute solution of $CaCl_2$, and at the center of a similar arc between carbon poles which had been soaked in a strong solution of the same salt. Since the exposure times for lines of equal strength in the two cases were in a ratio of about 1 to 10, it seemed to us justifiable to assume that the density of the luminous Ca vapor was markedly different in the two cases, and therefore that differences in vapor-density alone could not be responsible for the differences in wave-length observed between center and pole. It seemed fairer, on the whole, not to mention Royds's work at all than to offer this very limited evidence in refutation of it.

In his latest article[6] Royds finds the same wave-length for the green copper lines at the center of an arc between poles of pure copper and between poles of a 20 per cent alloy of copper. This seems to confirm our view. At any rate, I find it much easier to believe that a change in vapor-density alone cannot affect a change

[1] *Astrophysical Journal*, **43**, 161, 1916.

[2] *Ibid.*, **38**, 141, 1913.

[3] *Ibid.*, **42**, 231, 1915.

[4] *Kodaikanal Bulletins*, Nos. XXXVIII and XL.

[5] *Astrophysical Journal*, **43**, 161, 1916; **44**, 65, 1916.

[6] *Ibid.*, **45**, 112, 1917.

in wave-length than to assume with him that experiments like ours "fail or at any rate are inconclusive, because there is no reason to believe that the atoms have been separated to a greater distance apart with the smaller amount of material." While I might perhaps agree with him that "exposure times are not a sufficient test of vapor-density," I find it easier to believe that radiating calcium vapor is denser in one arc than in another when plates are secured with one-tenth to one-twentieth the exposure time, than to assume, as Royds does, that when a small amount of $CaCl_2$ is present in the carbons the atoms vaporize in clusters, and are not removed from each other's influence any more than when a larger amount has been used.

The very tentative nature of our suggestion that amplitude of vibration might be effective in disturbing wave-lengths will, I think, be recognized at once by anyone who reads our paper. It seems unwise to enter into a discussion of the fundamental causes of pole-effect until more facts are known. This feeling is confirmed rather than otherwise by Royds's latest paper, and one must regret that he feels it necessary to abandon for the present a field in which his contributions have been so valuable.

When we have considerably more data at our disposal it may be well to analyze the fundamental causes of such changes of wave-length as occur in pole-effect and pressure-shifts. It seems to me that any discussion of vapor-density as a possible cause should be considered in the light of Sir Joseph Larmor's[1] suggestion of a modification of the dielectric constant of the medium, or Humphrey's[2] suggestion of the mutual magnetic influences of neighboring atoms, or Stark's[3] suggestion of mutual electric influences. In the meantime I do not feel that our suggestion of the possible rôle of amplitude need be taken too seriously.

I do not agree, however, with Dr. Royds that rise in temperature is the "most effective and probably the only certain way" in which to increase the amplitude of vibration of the light-emitting electrons within an atom. In fact, it seems to me that in an electric

[1] *British Association Report*, p. 555, 1897.

[2] *Astrophysical Journal*, **23**, 233, 1906.

[3] *Jahrbuch der Radioaktivität und Elektronik*, **12**, 349, 1916.

arc or spark the velocity of impinging electrons and canal rays may be the important factor, and these velocities depend on potential gradients rather than on temperatures. These are certainly the predominant factors in vacuum tubes. Perhaps the only effect of temperature is to produce the Doppler broadening, which is, of course, symmetrical.

I agree with Royds that temperature alone does not appear to be the cause of pole-effect. But it seems to me inevitable that temperature must very largely control vapor-density, and the first argument which Royds uses to show that pole-effect is independent of temperature might be used as an argument that it is independent of vapor-density as well. To my mind at least, there is a much more direct relation between temperature and vapor-density in the arc than between temperature and amplitude of vibration.

However, I do not wish to be drawn into a further discussion of these matters at present. Many related facts will occur to anyone who has thought along these lines, but the time for serious discussion does not seem to me to have arrived. Certainly I have no favorite theory to defend, and shall be quite as pleased if the final outcome points to vapor-density as I shall if it points to any other factor; but I doubt that the explanation will be a simple one, or the same for all lines. In the final solution of these questions I am sure that the interesting and valuable contributions of Dr. Royds will have great weight.

HENRY G. GALE

RYERSON PHYSICAL LABORATORY
February 6, 1917

A VARIATION IN SOLAR ROTATION

A theory has been recently advanced by R. E. DeLury which would account for discrepancies in spectroscopic determinations of the solar rotation by varying quantities of atmospheric haze. In particular[1] he has applied this theory to some results given by the writer in a paper with the above title.[2] Owing to the pressure of

[1] *Astrophysical Journal*, **44**, 198, 1916.

[2] *Ibid.*, **43**, 156, 1916.

military duties, the writer is unable to obtain any experimental data on this interpretation of his results, but in the meantime the following comments may be made.

1. DeLury states: "The record shows that in general high values of the rotation in the observations mentioned were obtained on the brighter days and low values on the hazier days."

During the period under consideration, namely, June 21 to August 16, DeLury made three entries in the observation book with reference to haze:

June 21, "Bright"	$V=1.911$ (low).
July 11, "Very hazy"	$V=1.975$ (high).
Aug. 16, "Bright, some water-vapor haze"	$V=1.977$ (high).

In addition to this record the writer made a visual estimate of brightness on each day that plates were taken and entered it in the observation book:

5 represents a very brilliant day—rare in Ottawa.
4 represents a bright day—normal observing weather.
3 represents a day slightly hazy.

Plates taken on days hazier than 3.5 were *not* measured. On ten days of low values (1.846 to 1.919 km) the average brightness was 4. On fifteen days of high values (1.943 to 2.007) the average brightness was 3.7. In other words, the record, if it shows anything, shows that in general high values were obtained on hazy days and low values on brighter days, in direct contradiction to DeLury's statement above.

2. DeLury bases his discussion on plates made on two dates, June 24 ("July 13" evidently an error, since the writer measured no plates made on that date) and July 20, which are described as "Hazy" and "Bright," apparently from memory, as it is not so recorded in the observation book. The measures show evidence, according to the criterion of varying velocity for different line intensity, of about 8 per cent haze. This, on his own hypothesis, should produce about an 8 per cent difference in velocity on the two dates. The difference in mean velocity is actually 12 per cent, leaving a 4 per cent change in velocity to be accounted for by

sources external to the atmosphere, which, as far as it goes, may be taken as a confirmation of the writer's results.

These experimental data are, however, both meager and inconclusive. It seems unfortunate that, with so much material on hand for discussion, plates for only two days were measured. The results for these two days are, moreover, inconclusive, since the plates measured were taken two to three hours after the writer had completed his exposures. As a consequence, on June 24 DeLury's plates were taken through clouds whose presence he notes in the observation book, whereas the writer's plates, exposed earlier, were taken with a clear sky. Furthermore, the difficulties of measurement of broad lines increase the errors and chances of prepossessions, especially with an observer whose last published measures[1] show a probable error for even well defined lines of 0.06 km, i.e., of the same order as the differences measured.

TABLE I

Day	Interval	Time First Plate	Residual First Minus Last	Mean Velocity Day	Day	Interval	Time First Plate	Residual First Minus Last	Mean Velocity Day
		A.M.	km	km			A.M.	km	km
172....	60^m	6.50	+0.022	1.911	201...	101^m	6.24	−0.053	2.026
176....	73	7.41	+ .052	1.914	208...	62	6.11	+ .034	1.996
179....	57	8.04	− .043	1.866	210...	121	6.26	− .067	1.964
182....	32	7.52	+ .049	1.871	222...	88	8.45	− .024	1.966
185....	36	6.02	+ .056	1.893	228...	184	8.41	+ .064	1.977

NOTE.—In the last four entries in this table, where more than four plates were made in a day, the early and late plates have been grouped together and averaged to diminish accidental differences without appreciably shortening the interval.

3. The plates taken by the writer contain in themselves evidence that haze plays no part in his results. In order to obtain measurable telluric lines to correct for instrumental error, the plates were all taken in the early morning. Evidently, if haze was present in sufficient quantities to effect the results, it will show its greatest effect when the sun's light was passing through the thickest layer of atmosphere. That is, for plates effected by haze the residual first plate minus last plate should be negative. In Table I are given the residuals for ten days in which the first

[1] *Transactions Royal Society of Canada*, 1912, Sec. III, p. 1.

plate was taken before 9:00 A.M. and the interval was at least thirty minutes. A study of this table indicates that, for the first five days, when the velocities are lower than the mean, the residuals are preponderatingly positive instead of negative, as they should be if haze produced the low velocities. Similarly the last five plates of high velocity show, if anything, negative residuals, which would indicate the presence of haze.

This result is further confirmed by grouping the plates as in Table II. Here also the residuals are preponderatingly positive.

TABLE II

Groupings of Days	No. of Days	Average Values		
		Time First Plate	Time Last Plate	Residual First Minus Last
		A.M.	A.M.	km
Every day plates were made	25	7.35	8.818	−0.0016
Days first plate made before 8:00 A.M. ..	14	6.53	7.46	+ .0096
Grouping of Table I	10	7.16	8.30	+ .0090

Even the first grouping, which gives a small negative residual, is explained when it is remembered that on fifteen days included in that grouping the average interval between first and last plates was fifteen minutes—too small an interval to be rightly included. Removal of these days gives the last grouping, which is positive.

Summarizing: (1) There is no evidence in the observational record of the varying atmospheric haze required on DeLury's hypothesis to produce the variation. (2) His own measures of its presence are both meager and inconclusive. (3) Internal evidence from the writer's own measures does not reveal the presence of the haze.

In conclusion the writer wishes to emphasize the fact, which is also borne out by the preceding discussion, that in making exposures the greatest care was taken to avoid hazy days. Certainly plates were not taken on days with an 8 per cent to 10 per cent haze required by DeLury's hypothesis to account for the variation obtained.

H. H. Plaskett

Kingston, Ontario
December 1916

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